International conference on Graphic Engineering
June 19th – 21st, 2013 Madrid,Spain
INGEGRAF – ADM – AIP PRIMECA

Analytical studies of axial load transfert in an interference fit fastener assembly

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Article Information

Keywords:
interference fit fastener,
analytical study,
design of experiments,
slab method.

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Abstract

This paper proposes several analytical approaches in a study of interference fit fasteners. The influence of both the friction factor and the interference level on the loss of axial load is investigated. The first approach exploits Lame's equations as in previous studies while the second approach goes further by exploiting the slab method to obtain differential equations. The two approaches are evaluated and compared through a previously validated axisymmetric FE model using Abaqus.

List of symbols

d	External diameter of the screw
d_i	Internal diameter of the screw
D	External diameter of the plates
D_i	Internal diameter of the plates
E_i	Young modulus of the screw
Ee	Young modulus of the plates
h	Contact length between the screw and the plates
I(z)	Interference on diameters with tightening load at position z
L	Thickness of a plate
S	Tightening load applied to the nut
t%	Interference rate
T	Axial load under the screw head
ν_i	Poisson ratio of the screw
ν_e	Poisson ratio of the plates
f	Friction factor between the screw and the plates
Δ	Initial interference on diameters

1 Introduction

Bolt assemblies are a very common feature of engineering designs. When the bolt assembly is located in a critical sector, it may be replaced by an interference fit fastener. In that case, the screw contains a cylindrical part having a diameter larger than that of the hole where it is fitted and the two parts are locked together not only by the tightening load but also by the combined effect of friction and radial pressure due to interference in size at their interface diameter. It has been proved that this process protects the fastener hole from crack propagation [1] and several studies have shown that using interference fit fasteners increases the fatigue life of structures [2]. For these reasons, interference fit fasteners are often used in space and aeronautical applications. Duprat et al. [3] have studied fatigue life prediction for an interference fit fastener. They show that, in 90% of cases, crack initiation does not take place at the edge of the hole but rather between the bore and the edge of the test specimen.

Such screws can be pushed or pulled through the plates by means of specific tools (fig. 1).

Fig. 1 Interference fit fasteners and dedicated tool.

We can assume that, in some cases, particularly when the interference level and the friction factor are too high, a large part of the tightening load can be lost through the tangential contact forces at the interference diameter (fig. 2).

Moreover, a gap can occur between the screw head and the plate, creating an unauthorized defect. Of course, such a situation is not acceptable in an industrial application. In the preliminary design phase of such an assembly, it would be interesting to be able to analytically evaluate the axial loss of load as a function of the two main design parameters: the interference level and the friction factor.

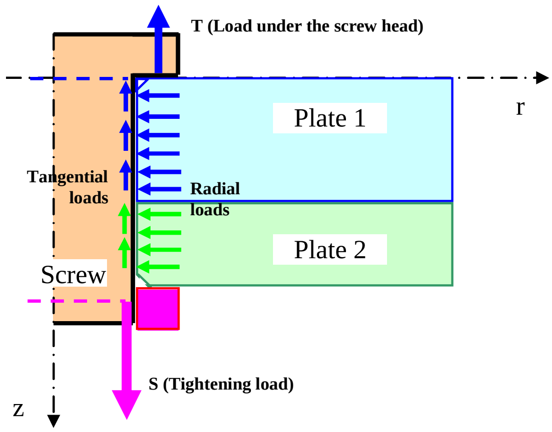

Fig. 2 Illustration of the axial loss of load.

The average pressure at the shrink-fit interface is commonly calculated by using Lame's equation [4, 5]. However, it has also been shown that an analytical study using Lame's equation can differ significantly from a finite element simulation [6].

Analytical approaches to asymmetrical forming processes like rolling and forging have been developed using the slab method [7-11]. This may be an interesting way to develop an advanced analytical approach for our study.

Paredes et al have also developed an axisymmetric finite element analysis dedicated to interference fit fasteners [12, 13].

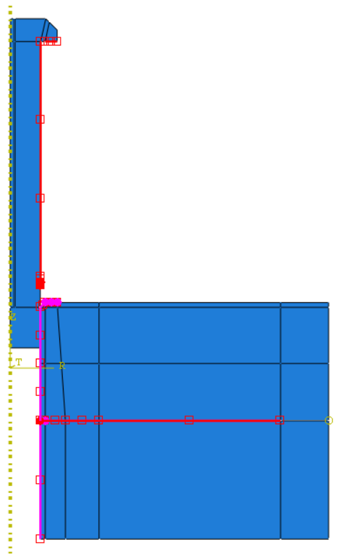

Fig. 3 Axisymmetric FEA.

This approach, illustrated in fig. 3, can be used as a reference with which the performance of an analytical approach is compared.

The aim of this paper is to define and evaluate methods of determining the axial load in interference fit fasteners, which could be exploited in preliminary design.

A simplified analytical model based on Lame's equations is presented in section 2. Then a more advanced analytical model is described in section 3. A case study enables both the analytical models to be quickly compared with a finite element analysis in section 4. To evaluate the performance of the advanced analytical approach more precisely, section 5 is dedicated to a study based on the design of experiments.

2 Simple analytical model

The idea developed here consists of superposing a screw in only a uniform tension state and a screw with only an interference state (fig. 4).

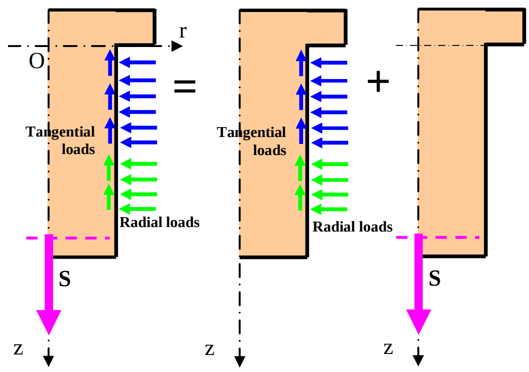

Fig. 4 Load decomposition.

To model the behaviour of an interference fit fastener, the easiest technique is to consider cylindrical parts of similar length where the inner cylinder fits completely inside the outer cylinder. When a situation with no axial stress is considered, a plane strain state can be assumed with a constant radial pressure along the whole contact length. This assumption can be exploited for thick parts with no axial stress.

Let us consider the case of a tube subjected to both internal and external pressure as illustrated in fig. 5.

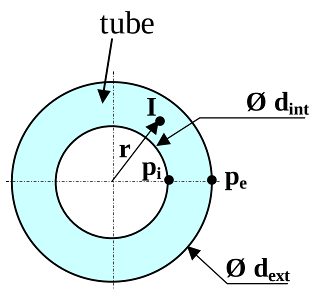

Fig. 5 Equilibrium of a tube subjected to pressure.

It is possible to calculate the radial displacement $u_{I(r)}$ of a point I situated at a radius r by using the following equation:

$$u_{I(r)} = \frac{1-\nu}{E} \frac{d_{\text{int}}^2 . p_i - d_{ext}^2 . p_e}{d_{ext}^2 - d_{\text{int}}^2} \cdot r + \frac{1+\nu}{E} \frac{d_{\text{int}}^2 . d_{ext}^2 . (p_i - p_e)}{4.(d_{ext}^2 - d_{\text{int}}^2) . r} \quad (1)$$

This equation can be applied to the screw in order to identify the radial displacement in the contact area under the effect of the contact pressure p. In the case of a screw with an axial hole, the following equation is obtained:

$$u_i = -\frac{1-\nu_i}{2.E_i}\frac{d^3.p}{d^2-d_i^2} - \frac{1+\nu_i}{E_i}\frac{d_i^2.d.p}{2.\left(d^2-d_i^2\right)} \qquad (2)$$

Eq. 1 can also be exploited to consider the radial displacement of a plate in the contact area under the effect of the contact pressure p.

$$u_e = \frac{1-\nu_e}{2.E_e}\frac{d^3.p}{D^2-d^2}\cdot r + \frac{1+\nu_e}{E_e}\frac{d\ .D^2.p}{2.\left(D^2-d^2\right)} \qquad (3)$$

Both the radial displacements have to consider the initial interference. Thus:

$$\frac{\Delta}{2} = \mathrm{u_e} - \mathrm{u_i} \qquad (4)$$

These three equations (2, 3, 4) enable the radial pressure to be calculated as a function of the interference:

$$p = \frac{\Delta}{\frac{d}{E_e}\cdot\left(\frac{D^2+d^2}{D^2-d^2}+\nu_e\right)+\frac{d}{E_i}\cdot\left(\frac{d^2+d_i^2}{d^2-d_i^2}-\nu_i\right)} \qquad (5)$$

When a constant pressure is considered throughout the contact area, the evolution of the axial load on the screw can be evaluated at any axial position:

$$F(z) = S - d.\pi.p.f.(h-z) \qquad (6)$$

with $z = 0$ under the screw head and $z = h$ at the contact with the nut. (See fig. 2 and fig. 4.)

In this simple analytical model, a constant pressure is considered. However, when an axial load is applied to the screw, the effect of the Poisson ratio reduces the screw's external diameter. This causes the radial pressure to be modified. We thus propose another approach that takes the effect of the Poisson ratio into account.

3 Advanced analytical model

The previous studies detailed in the introduction show that basic models based on Lame's equation have only an average ability to determine the contact pressure. In order to be more precise, we now try to develop a more advanced analytical approach based on the study of thin slabs [7-11].

This approach considers several hypotheses:

1. Parts have a basic axisymmetric geometry
2. The friction factor is constant
3. There is no shear stress from within a slab

In this approach, we try to take into account the effect of the axial tension on the radial pressure. Two tubes of equal length are considered. Axial stress in the screw is taken to be constant in any section normal to the axis (axial stress is only axial position dependent).

Let us consider that the screw is first introduced into the plates with a given interference value Δ and then that an axial load S is applied.

Studying the equilibrium of a thin slab of the screw (fig 6) leads to eq. 7.

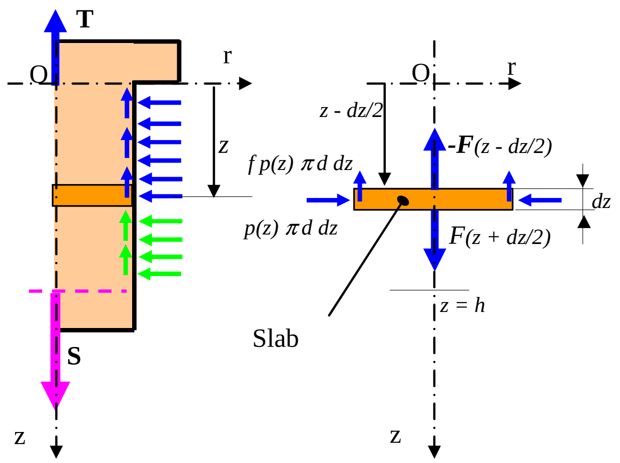

Fig. 6 Study of a thin slab

$$f.p(z).d.\pi.dz + F\left(z-\tfrac{dz}{2}\right) = F\left(z+\tfrac{dz}{2}\right) \qquad (7)$$

p(z) can then be evaluated

$$p(z) = \frac{1}{d.f.\pi}\frac{dF(z)}{dz} \qquad (8)$$

The axial load *F(z)* causes the screw to lengthen and, because of the Poisson Ratio, its diameter is reduced as described in eq. 9.

$$\delta(d) = -\upsilon_i.\frac{4\cdot d}{E_i.\pi.\left(d^2-d_i^2\right)}.F(z) \qquad (9)$$

The initial interference is thus reduced when the axial load is applied:

$$I(z) = \Delta - \upsilon_i.\frac{4\cdot d}{E_i.\pi.\left(d^2-d_i^2\right)}.F(z) \qquad (10)$$

Eq. 5 gives the pressure *p(z)* corresponding to the real interference *I(z).*

$$p(z) = \frac{I(z)}{\frac{d}{E_e}\cdot\left(\frac{D^2+d^2}{D^2-d^2}+\nu_e\right)+\frac{d}{E_i}\cdot\left(\frac{d^2+d_i^2}{d^2-d_i^2}-\nu_i\right)} \qquad (11)$$

Eqs. 8, 10 and 11 can be combined to obtain the following differential equation:

$$\frac{dF(z)}{dz} + K_1.F(z) = K_2.\Delta \qquad (12)$$

where

$$K_1 = \frac{\left(\frac{4.d.f.\upsilon_i}{E_i.\left(d^2-d_i^2\right)}\right)}{\frac{1}{E_e}\cdot\left(\frac{d^2+D^2}{D^2-d^2}+\nu_e\right)+\frac{1}{E_i}\cdot\left(\frac{d^2+d_i^2}{d^2-d_i^2}-\nu_i\right)} \qquad (13)$$

and

$$K_2 = \frac{f.\pi}{\frac{1}{E_e}.\left(\frac{d^2+D^2}{D^2-d^2}+\nu_e\right)+\frac{1}{E_i}.\left(\frac{d^2+d_i^2}{d^2-d_i^2}-\nu_i\right)} \quad (14)$$

Eq. 12 can be solved to calculate the axial load in the screw at any axial position *F(z)*:

$$F(z)=\left(S-\frac{K_2}{K_1}.\Delta\right).e^{K_1.(h-z)}+\frac{K_2}{K_1}.\Delta \quad (15)$$

The axial load under the screw head *T= F(0)* can be evaluated:

$$T=F(0)=\left(S-\frac{K_2}{K_1}.\Delta\right).e^{K_1.h}+\frac{K_2}{K_1}.\Delta \quad (16)$$

4 Comparison of the two analytical approaches on a case study

In order to illustrate the formulae obtained and compare the efficiency of the two analytical approaches, a case study is detailed. The analytical results are also compared to a finite element analysis performed with Abaqus as proposed by Paredes [12, 13].

The case study is defined by the following data. The screw is made of titanium alloy (E = 110 GPa, ν = 0.32). Its radius is 6.35 mm and the screw length is 70 mm. The two plates are made of aluminium alloy (E = 72 GPa, ν = 0.35) and are 25 mm thick with an external radius of 60 mm. The interference rate is *t*% = 0.8%. The friction factor is equal to 0.04.

The interference rate is defined by the following equation:

$$t\% = \frac{d-D_i}{d} = \frac{\Delta}{d} \quad (17)$$

The finite element analysis with Abaqus comprises several steps:
1. Introduction of the screw by moving its lower end
2. Application of the introducing load to the thread of the screw
3. Removal of the introducing load
4. Application of the tightening load to the thread of the screw

Using Abaqus, the axial load can be calculated at any axial position by summing the tangential contact loads. The axial load can also be evaluated by using eq. 6 for the basic analytical model and eq. 15 for the advanced model.

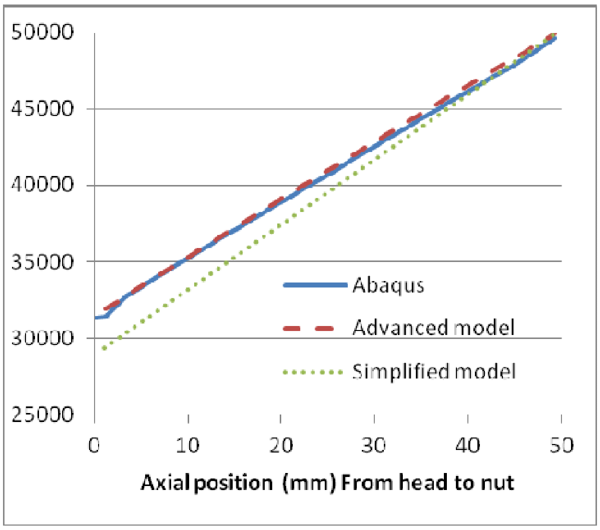

Fig. 7 Axial tension of the screw (N).

In the case presented, fig. 7 shows that the advanced model gives results very close to the finite element analysis. As expected, the basic model only gives an approximation of the results given by the finite element analysis considered as our reference.

The advanced model also enables the evolution of the real interference state to be evaluated along the whole axis (fig. 8). This could not be easily extracted using Abaqus.

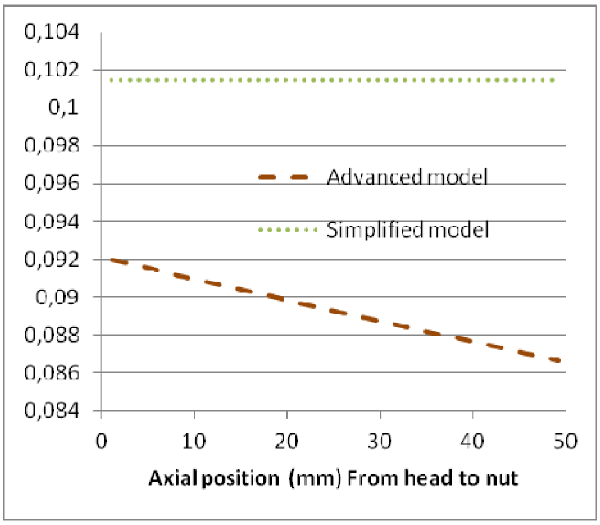

Fig. 8 Interference level considering the axial loads (mm).

We can note that the interference rate tends to fall when an axial position close to the nut is considered. This is altogether logical as the closer the axial position considered is to the screw head, the more the axial load is increased.

Moreover, the maximum interference level obtained with the advanced analytical approach (0.093 mm) remains lower than the initial interference level considered in the simplified analytical approach (0.101 mm). The axial tension in the screw thus has a significant effect on the results. This explains the deficiencies of the basic analytical approach, which neglects the combined effects of the axial load and the Poisson ratio.

Finally the radial pressure obtained by the three approaches can be compared (fig. 9).

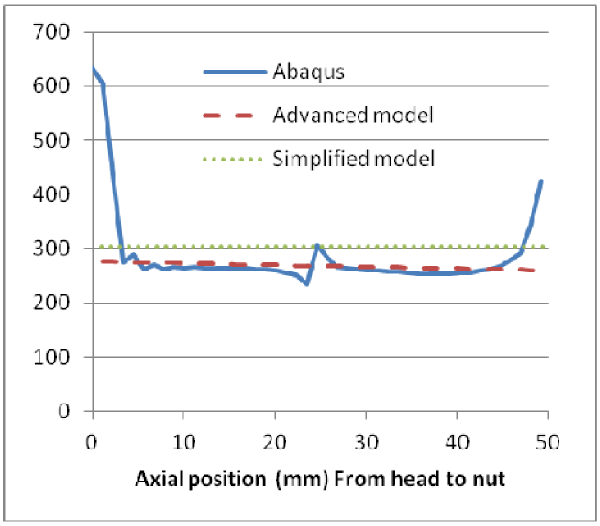

Fig. 9 Radial pressure (MPa).

The results presented in fig. 9 clearly show that the advanced analytical approach gives a good estimate of the radial pressure when the end effects are neglected whereas the simplified approach overestimates the mean radial pressure.

The advanced model could be efficient for long screws where the end effects can be neglected.

5 More precise evaluation of the advanced analytical model

The proposed advanced analytical model is now evaluated as a design area that fits the industrial application domain where it could be exploited. The idea is to compare the main results: the loss of load under the screw head and the loss of load between plates, by using them in a preliminary design stage. In this case, the finite element analysis is considered as our reference.

Material data is considered as constant here:

The screw and the nut are made of titanium alloy (E = 110 GPa, ν = 0.3). The two plates are made of aluminium alloy (E = 72 GPa, ν = 0.33).

In agreement with our industrial partner, 5 parameters, considered as significant, are investigated in 2 possible states. This defines the design area:

-screw diameter, *d,* = 6.35 or 12.7 mm,
-interference rate, *t%,* = 0.8% or 1.2%,
-plate thickness, *L,* = 6 or 24 mm (each),
-external diameter of the plates, *D,* = 2*d* or 10*d*,
-friction factor, *f,* = 0.02 or 0.06.

The tightening load is considered as directly dependent on the screw diameter and is taken to induce a stress equal to half the ultimate tensile strength (895 MPa). Thus *S* equals 15000 N when *d* = 6.35 mm and *S* equals 60000 N when d = 12.7 mm.

The other geometrical properties (e.g. lengths of the nut or screw head) are chosen to be close to the values that can be encountered industrially. For example, the screw head radius is 5.5 mm and the head is 2.5 mm thick when *d* equals 6.35 mm and those values increase to 10 mm and 5 mm, respectively, when *d* equals 12.7 mm.

Considering the 5 parameters and 2 levels could involve 2^5 = 32 experiments.

As the finite element analysis on Abaqus has to be modified by hand, a smaller study is considered. It is described in table 1.

Experiment number	Factor 1 (*d*)	Factor 2 (Δ)	Factor 3 (*L*)	Factor 4 (*D*)	Factor 5 (*f*)
Fr1	-	-	-	+	+
Fr2	-	-	+	+	-
Fr3	-	+	-	-	+
Fr4	-	+	+	-	-
Fr5	+	-	-	-	-
Fr6	+	-	+	-	+
Fr7	+	+	-	+	-
Fr8	+	+	+	+	+

Tab. 1 Detail of the experiments.

Table 2 gives the results obtained.

Experiment number	T (N)			Load between plates (N)		
	Advanced analytical	Abaqus	gap %	Advanced analytical	Abaqus	gap%
Fr1	11068	11090	-0,20	13058	13071	-0.10
Fr2	9713	9741	-0.29	12400	12423	-0.19
Fr3	10406	10685	-2.61	12724	12870	-1.14
Fr4	8837	9164	-3.57	11956	12150	-1.60
Fr5	58110	58186	-0.13	59057	59129	-0.12
Fr6	36552	38322	**-4.62**	48488	49561	-2.16
Fr7	55901	55733	0.30	57955	57911	0.07
Fr8	8516	8521	-0.06	34892	34907	-0.04

Tab. 2 Results of the experiments.

The results presented in table 2 show that, in most cases, analytical results are very close to those obtained by the finite element analysis as the difference is less than 1% in 10 out of 16 cases.

The maximal difference is about 5%. This is an acceptable accuracy in a preliminary design context. The advanced model is thus very interesting for the evaluation of the loss of load under the screw head and under the plates.

6 Conclusion

This paper investigates two analytical studies describing the loss of load along the axis of interference fit fasteners. The first approach exploits Lame's equations to give a first level approximation. To go further, the slab method is exploited to obtain differential equations that are solved analytically.

Both models are compared to the results of a finite element study performed with Abaqus. As expected, the advanced model, based on the slab method, gives very interesting results.

Thus, this approach can be used in the preliminary design step to evaluate the best design for a given application.

It could be interesting to enhance this approach by including the edge effects to give more accurate results.

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