



Freeform surfaces adaptation through developable strips.

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Abstract

There has been in recent years a significant increase in the use of free forms, double curved forms in the field of Architecture. This development has been linked to advances in digital manufacturing and representation systems. When it is about to build complex forms must find rational construction systems where the economy is present without forgetting means fidelity in reproducing the form as designed.

An adequate strategy for the development and construction pass through their adaptation by using developable surfaces.

This involves a process of adaptation and simplification of a double curved freeform surface by a developable surface or single curvature surface. Here we propose a system that is based on the apparent contours on a surface for an adaptive approach to freeform surfaces by developable surfaces.

1 Introduction

Architecture has seen in recent years an extensive use of double curvature surfaces. Simultaneously, supported by digital media design and computer aided manufacturing has been an important development of building systems such this kind of surfaces, as we can see in Frank Gerhy Architecture related with Glymph and Shelden works [1,2].

There has been an explosion in the control systems of the form, not only from the point of view of CAD technology software but through very interesting theoretical developments as those published by Pottman [3] from that Architecture has taken advantage by offering amazing results.

Using double-curved surfaces is not limited exclusively to architecture but has been part of the record of the geometry of engineering in general, such as automotive or naval construction is where the architecture has taken inspiration.

Many industrially made materials are approximately unstretchable, metal sheets, plywood or glass have been widespread employed at engineering and building construction. So there is no doubt about the advantages offered, mainly from the economic point of view, the processes that use flat or developable surfaces in the resolution of doubly curved ones.

Through two prototypical developable surfaces has been built an adaptation method of double curvature surfaces using developable surfaces.

Using the concept of geometric apparent contours and through the systematization of a process inspired by traditional projective geometry is built an algorithm, using the newest and outstanding software that systematizes obtaining single-curvature strips or flat facets strips. In order to be able to address its construction using materials with the possibility of bending in one direction or rigid material with no possibility at all of being bent by simple procedures.

These strips are from the geometrical viewpoint absolute developable as cones or cylinders.

2 Approaching and Geometric Basis

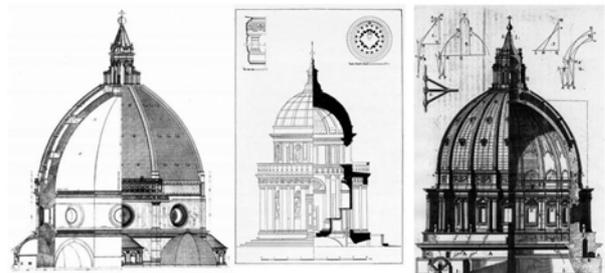


Fig. 1 Classical spherical domes.

At any of the numerous examples of spherical domes (Fig.1), the sphere shape is built by cylindrical sections actually, basic geometry of the dome, are supported on said surface along its meridian curves. These cylindrical sections and circumscribed developable tangent surfaces to the original spherical surface, two by two cylindrical sections intersecting along a curve outside the primitive surface, constructively materializing in said dome ribs, forming the border of the cylindrical surface of the panel.

Analogically to this cylindrical adaptation, we could involve the primitive spherical surface using conical sections and therefore the developable surfaces will go to produce conical projection contours, between the apparent contour and the intersection of the adjacent projection surfaces or their prolongations. This surface will always be the outer skin of the sphere.

We can therefore conclude that we can wrap a spherical surface with a family of developable surfaces, cones or cylinders, which are supported on this surface tangentially along apparent contour lines, which in this case are the sphere major circles.

It performs an adaptation process of the double-curved surface due to constructive request. Previously occurs a geometrical simplification the double curved surface is adapted using a set of single-curved surfaces, so developable from the geometrical point of view. These cylindrical or conical sections can be understood as a series of strips, a envelope of the spherical shape.

The method we propose is inspired by this principle which we well know. Abstracting the process that produces the geometrical approximation of the spherical dome, it could be repeated at any surface regardless of its geometry, we want to solve evidently the case of double curvature surfaces and the spherical geometry is one of them.

Thus we can draw on any surface several group of lines, apparent contours along that we can built tangent developable surfaces, at this case cones or cylinders, over the surface itself. And in this way extract a section delimited from these developable surfaces, similarly to what happens to the spherical dome, the intersection of the auxiliary cones or cylinders determining the limits of these.

To draw the apparent contour over free surfaces could be impossible without using CAD software. In our case we used the modeling tool Rhinoceros [4]. It might be possible to use mathematical differential methods, but the way we work is intended as a systematization of the mechanisms used in projective geometry. This approximation is carried out by using an algorithm that systematizes the process that theoretically could be addressed manually. Translating the operating structure and proceeding to build the same in Grasshopper [5]. Grasshopper is powerful generative design software; it is a *plugin* that uses Rhinoceros as graphical support.

2.1 Apparent Contours.

When we draw a surface, using any system of representation, either a freehand drawing on paper or software-rendered model, the process needs to define a line that marks the edge of the object represented, separating the part of the object that the observer is able to see, which is hidden. This is basically the apparent contour of a surface.

Geometrically, following the definition of *Izquierdo Asensi* [6]: "*the apparent contours of a surface Σ from a point V outside it, σ is the locus of contact points A, B, C, \dots tangent to or belongs to tangent planes to Σ passing through V .*" We also know that if V is an improper point, the cone becomes a cylinder.

A surface surrounding the object to be displayed, depending on the viewpoint V , the surface tangent to the object along a series of points that define their apparent contour is a developable surface, in this case a cone or a cylinder depending on the relative position of point V .

The concept is very simple, yet obtaining these apparent contours is difficult especially for models far from simple primitive forms. Even for these primitive simple shapes, obtaining shadow boundary lines, commonly applied in geometry of the apparent contours is manually laborious. The CAD software, simplify and improve this work by providing powerful tools in this regard.

We distinguish two common types of projection, cylindrical projection and conical projection. From one surface we obtain an apparent contour from cylindrical projection, projecting lines parallel to one another tangentially to that surface.

The set of parallel directions forms a cylindrical surface that circumscribed to the surface. The set of points of tangency between the two surfaces, base surface and projecting surface constitute the apparent contour of cylindrical or parallel projection. (Fig.2)

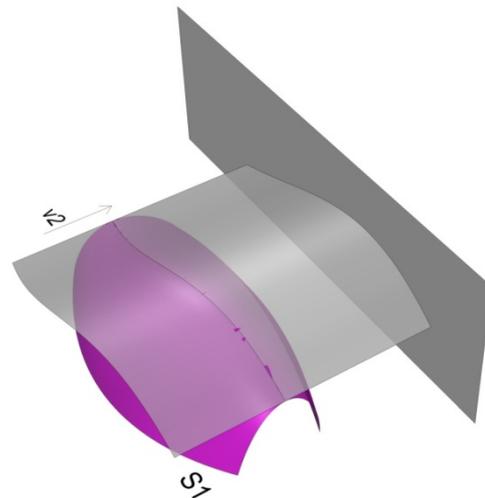


Fig.2. Projected cylindrical surface.

In similar way a conical projection consists of drawing several lines radiating from a point of view and touch tangentially on the surface by determining a curve whose conical surface of contact with the surface is the apparent contour therefor from its conical projection. (Fig.3)

This concept is used as noted in obtaining the shadow lines from volumes, being the apparent contour which separates the highlights and shadow areas, the cylindrical projection associated with lighting that produces a focus at infinity, where vectors are used to determine the ray parallel or projection lines. The central projection is about enlightenment through a focus from which depart own a bundle of lines surrounding the object illuminated by a conical surface.

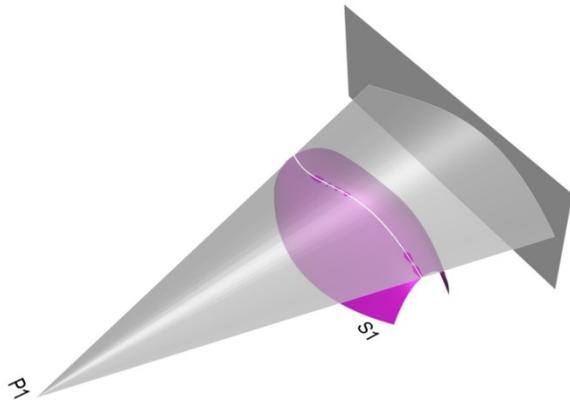


Fig. 3. Projected cylindrical surface.

We see that these lines have a sense beyond the pure surfaces that define playing an important role in the perception of the volume and the shape of the mentioned way of vision.

Clearly we get infinite results simply by varying the point of view or the projection direction such that on the surface appear in theory a boundary and apparent contours associated with each projection point of view or every variation of the projection direction.

Thus we obtain a particular curve on a surface, their apparent contour generated by circumscribing a plane tangentially to it, so that the tangents to the surface are the projection directions besides forming the generatrices of the developable surface projection, in this case cone or cylinder.

The apparent contours and apparent generators are used in the reconstruction of volumes based on planar imaging, using epipolar geometry. It is also the basis of performance of computer vision. Cipolla and Giblin [7]

We simply need to observe the possibility of perceiving objects and rebuild thanks to these contours. Further development wants to be an application of this concept finding to adapt freeform surfaces through on their apparent contour curves.

3 Methodology

3.1 Adaptation by cylindrical strips.

Taking a elliptical points or positive gaussian curvature free surface (Fig. 4), we proceed to generate a family of apparent contours, by moving the point of view along an improper line. This process is realized by obtaining the apparent contour of the surface from a series of points on a circle of infinite radius.

Obtaining the intersection of the projecting tangent cylinders to the base surface along adjacent contours, the mentioned projecting cylinders delimited between two intersections. Each of these parts of the original projecting cylinders contains both intersections and the tangent curve between the surface and the cylinder base, i.e. the apparent contour. (Fig. 5)

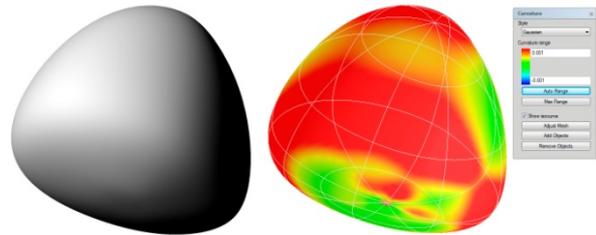


Fig. 4. Gaussian curvature analysis of a freeform elliptical surface.

On each of the apparent contours the cylinder generatrix parallel to the projection direction make this cylinder by slide.

We delimit a segment of each generatrix of the cylinder, the intersections of this cylinder between adjacent apparent contours, we define the strip through its generatrices.

The definition obtaining with Grasshopper allows construct systematically the process. Obtaining the contours, the contour selection and use of its later in the generation process strips.

At a later stage these strips will underpin a process of physical prototyping, developing a binding systems model. The procedure ensures strips developability due to its own generation process.

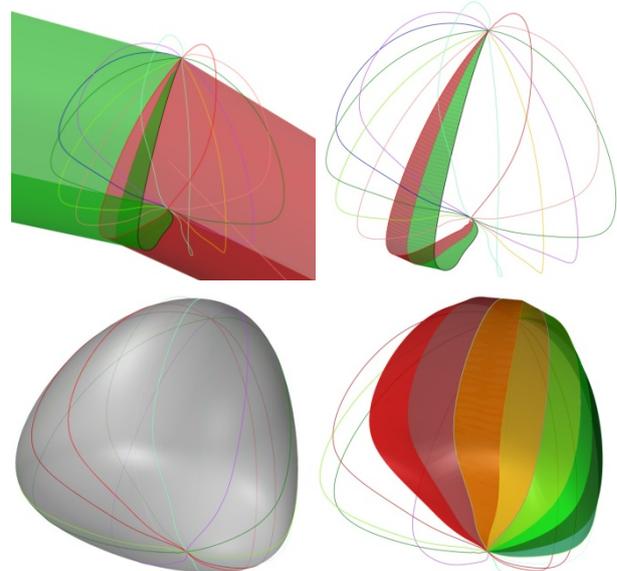


Fig. 5. Development stages sequence.

Proceeding in analogous way to that explained for elliptical surfaces, we trace an apparent contour family using a straight improper path from a several viewpoints situated at infinity, we can apply the method over a negative gaussian curvature free surface (Fig.6).

The path used in this case is external to the surface so that, as can be seen, no nodes appear thereon. The contours are not cut. Generating apparent contour curves along improper projection produces a series of contours that match the tangent curve of the surface considered and the cylindrical surface of projection.

Based on two contiguous edges that we know the project direction through which they have been generated, we perform the extrusion of the contour curve on surface in the project direction. Repeating this process for both curves, we obtain the auxiliary cylindrical surfaces.

If we intersect both cylindrical auxiliary surfaces, we obtain the trace belongs to both projection cylinders. This intersection as in the case of the elliptical surface is external to the base surface.

We proceed to divide each of the apparent contour curves in a certain number of points and drawn lines from them determined by the vector direction and come to cut the curve intersection of the generator cylinders. So we obtain a series of segments of the generatrices of the projecting cylinders limited by the apparent contour curves and the intersection of the cylindrical surfaces.

Each strip is obtained by generating transition between generatrix segments. We can approach as much as we want to surface dividing the curve in such a large number of points as desired for plotting segments. However, the surface from the strip is taken is fully determined by the curve and the projection direction that coincides with the generatrices of the cylinder.

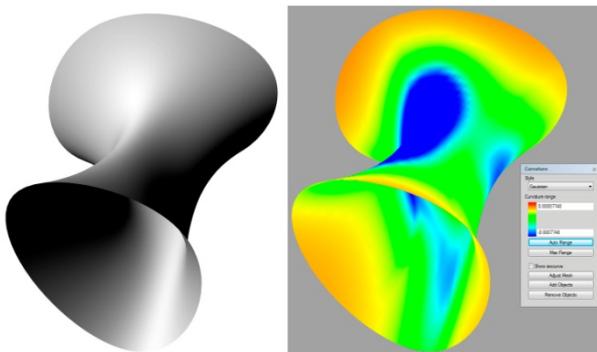


Fig. 4. Gaussian curvature analysis of a freeform hyperbolic surface.

For limited areas and warped edges, not flat curves, auxiliary cylindrical surface that rests on the apparent contour lines does not fit perfectly on the edge of the primitive surface. In some cases not reach it and in other ones exceeds this edge, so that a series of patches appear on the adapted surface edge. This requires a retouch process at the ends of the cylindrical strip surfaces obtained.

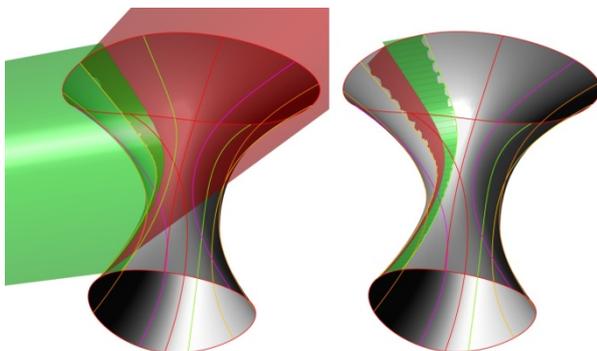


Fig. 7. Projection of cylinders supported in selected contours.

This problem obviously do not arises when we work with closed surfaces as we can see in the case of positive curvature surfaces.

The most direct way to address this problem would be extending the edges of the surface and then proceeded to cut the strips used as the limit of the original surface.

For an approach to form, to perform a rapid prototype of the surface, is would not be a biggest problem this drawback that we noted, however, we know that both curves of intersection of the two surfaces have the same length, the curves of intersection of two surfaces have isoperimetric contour development. This fact does not occur with the generation of cylindrical strips in this case since the cylindrical section obtained does not cover the original surface entirely.

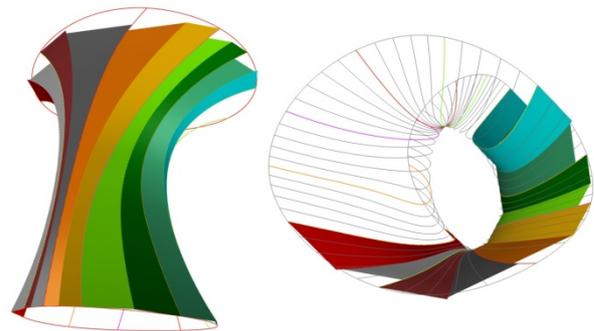


Fig.8. The limitation of the cylindrical strip at the open end of the hyperbolic surface.

3.2 Adaptation by conical strips.

An approach can perform over the same elliptical, positive curvature surface, this time using as support a conical contours family drawn on its surface.

The contours are generated by moving the viewpoint along a proper straight path that in this case due to a design decision, will cross the surface like an axis.

This condition causes the contours family is formed by several warped curves on the surface without tangents or cuts between them. The cutting points between the trajectory and the surface become so-called punctual contour points that tend toward the contours as the vertex of the projection cone approaches the surface. Towards this point converge the cutting point of the path, the cone vertex and the contour itself that at the limit it transforms at that point.

The view points in the same way that the trajectories are arbitrarily chosen by a subdivision of the path or conversely, we can obtain the paths that pass through certain points on the surface, searching for the intersection of the surface tangent plane at that point and the line used as the path.

Thus we can select various specific contours passing through certain points that offer singularities on the surface or it can interest in any design motif. Could likewise to make the surface section through the plane containing the path and divide the section produced on this surface. By plotting the tangents at the division points

of the intersection we obtain along the trajectory the vertex of the cone generated by the contour passing through these points.

The obtaining strips process is similar to cylindrical contours above. The cones that rest on adjacent contours have an intersection. There are actually two types of intersections, as we will show clearly on the hyperbolic surface. A first type superimposed cones is generated, for which, and as shown in the illustration (Fig.9), the lower cone must be homothetically scaled from the vertex to the intersection obtained sought. The other is generated by opposing cones.

The intersection of the two cones, external to the surface considered, together with the two contours which belong to the surface and each of the respective cones, determine a conical strip.

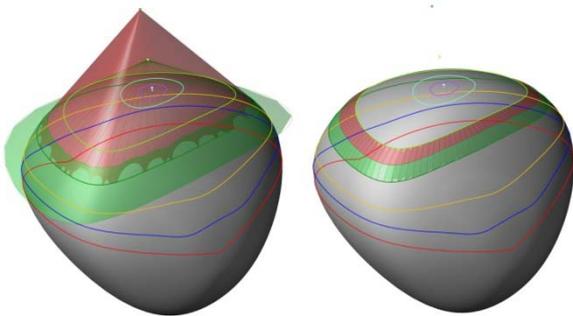


Fig.9. Projection of cones supported in selected contours.

From each cone obtained we extract two strips logically belong to the same conical surface, a contour delimited by the intersection of the cone and the immediately projecting cone with the lower and upper.

In the physical construction, these two strips that are generated, are merged into a single, differentiated at this point in the process looking for graphic clarity.

On each of the apparent contours the cone generatrix convergent in the point of view at the trajectory, generates the projection cone.

If we delimit a segment of each generatrix of the cone, between the intersections of the adjacent cones we define the strip through their generatrices.

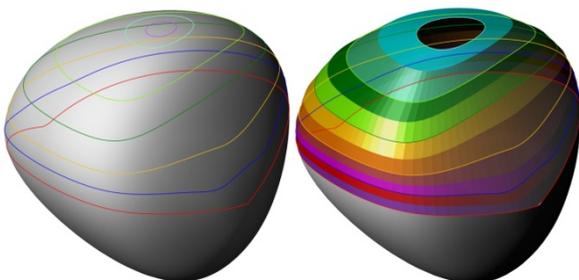


Fig.10. Approach by conical strips result.

As we can see the choice of the projection kind path and its relative position generates strips which differ substantially, not only in the morphology of the strip itself obtained, but the appearance and the interpretation of the object surface itself adaptation. The versatility of the

method in the treatment of surfaces offers the possibility of addressing the modeling of the same with a freedom that has a positive effect on the original surface interpretation, expanding the design possibilities.

Explain below the method of generating conical developable strips supported apparent contour curves generated by displacement of the point of view along a proper line on a negative curvature surface, hyperbolic surface.

The process is basically the same as that explained for conical strips. auxiliary cones path for elliptical points surface, obtaining the intersection thereof and delimitation of the conical surface between the intersection of the cones and corresponding contours apparent.

In this case we make concur this axis with a line passing through the throat of the base surface. The chosen axis not generate nodes and edges are not cut.

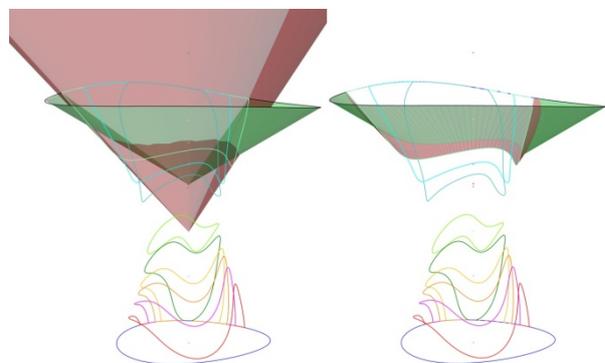


Fig.11. Projection of cones along proper axis.

The variation of the approximation axis and the position of the point of views along said axis or path is parameterized by the application of a script, the apparent contour generation process is automatic so that the different options can check and find the results in real time. The apparent contours on the surface, the free form we want to adaptate, that are suitable for the design can be selected chosen from a large number of possibilities that depends on varying viewpoints and the distance between them.

Thus we take advantage of one of the fundamental qualities of parametric design. One logical construction, the same process with different start values yield different results. So can be on designer hand to choose the option that is most appropriate to the particular project based on the requirements that he assigned.

Also as noted, these apparent contour curves have great expressive power so that the choice of apparent contour groups is linked to the design process, this is another quality that offers this method. Unlike the divisions that rely on other mechanisms, it has been found the appearance of significant and singular results with a visual interest and can contribute to the enrichment of the shape through a correct decision.

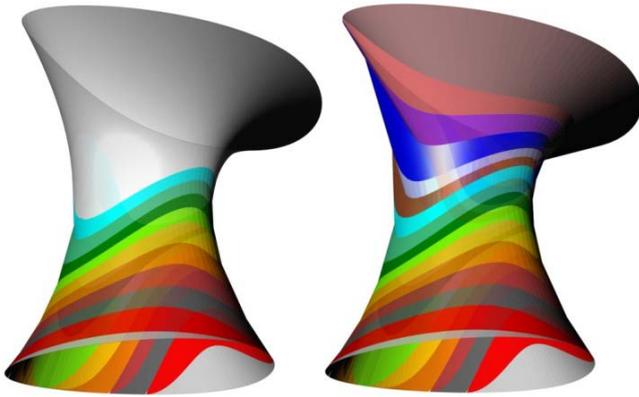


Fig 12. Conical developable strips from hyperbolic surface.

We exposed the method where the curvature surfaces is approximately uniform. The usual case however in presenting complex surfaces or freeform surfaces what is the one where the curvature is not the same sign.

The application of the method in this case varies substantially not exposed to the surface where the curvature is mainly positive or negative.

The strategy focuses on achieving an appropriate projection path, that yields results appropriate boundary apparent contour curves that facilitate the construction of developable strips.(Fig.10)

Again the process is performed by approximation, empirically, aided by an agile and versatile apparent contours extracting system, we can get several options, from multiple directions, choosing the most appropriate and within the contours selected more favorable.

We must mark the appearance of a node surface point where all the paths intersect. This node is determined by the parallel plane containing the circular path and is tangential to the surface. Where the surface is closed at the other end a new node must appear. The tangent plane to the surface at the second node would logically parallel to the first and to the plane containing the path.

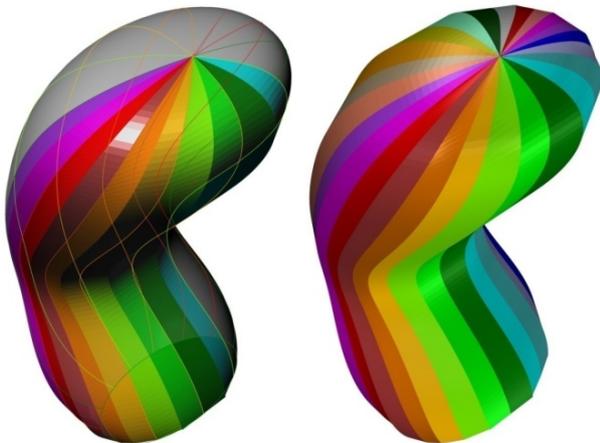


Fig. 13. Developable strips over a generical freeform surface.

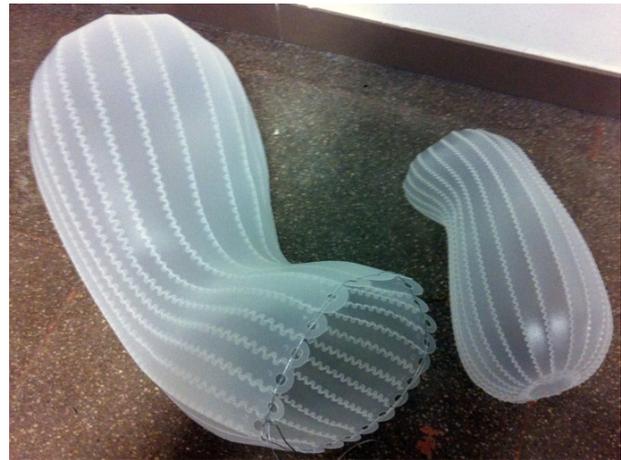


Fig. 14. Physical model built from polyethylene sheet

The method has been tested on various surfaces where the heterogeneity of the curvature is noticeable to the naked eye. It has resorted to the use of proper and improper straight paths or straight directions.

The basic objective of the research is to find results that can be extrapolated to the physical environment and constructive so we have proceeded to physically check the reality of the system, the approximation of the shape and behavior of the material.

All cases have been studied graphic and physically, we have been built prototypes using different materials. In all of the faithfulness can be seen in the physical reproduction of the result compared with the model. (Fig.14)

The construction was carried out using a laser cutting machine EPILOG 36EXT Legend - Model 9000, which allows fast cutting directly from the design program. It thus anticipate the difficulties that may arise due to material nature, thickness, tolerance in the cut, etc.. The limitation is set by the cost system that reduces the types of material on which it has been experience, reducing these to cellulosic sheets, paper, cardboard and plastics that are not chlorine compounds, mainly polyethylene and polypropylene.

The design, adaptation and operation of assemblies needed to join developable strips require a comprehensive study and development. It requires a much deeper investigation which introduced both computationally and manufacturing, material properties and structural behavior, checking the stress states generated in a timely manner edges, adaptation to the curvature of the surface, etc. similar to that described by Axel Kilian [8].

4 Conclusion and Future Works

Starting from the initial premises, constructive economy and fidelity in the reproduction of the form, we conclude that we have constructed a system adapting double curved surfaces through developable surfaces, that is intended to be extended to apply free-form surfaces.

The above forms the basis for the formulation of a geometrical method as has been demonstrated that provides satisfactory results, resulting suitable for the stated objective, the graphical shape adaptation and construction of prototypes as both finished elements that satisfy the two premises made.

Through the use developable strips that have been extracted from the cylinders or cones that are tangents along the apparent contours. This procedure is applicable when using a material that allows bending, obviously in a single direction. Therefore we are setting a geometric system that underlies the construction of double curvature surfaces with metal sheets and products of similar behavior.

The proposed method allows the geometric development of the manufacturing and prototyping methods, using production tools that have been tested successfully in prototype construction.

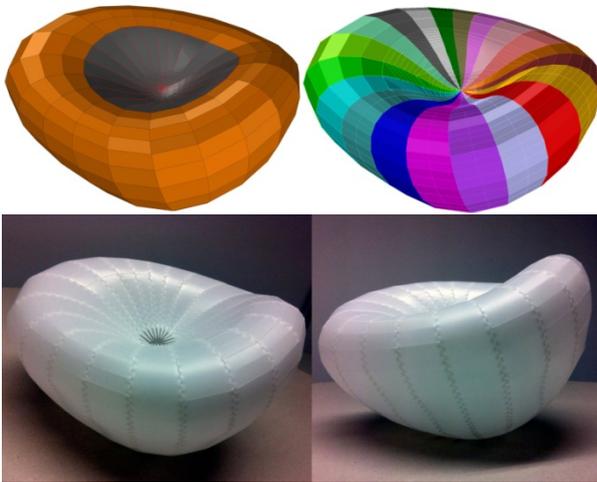


Fig. 15. Physical model built from polypropylene sheet

We speak about prototypes not models, in the prototype we can anticipate the behavior of materials in relation to the way they are arranged. The prototype has a value in relation to purely formal model, we have seen the results in terms of tolerances, movement, adaptability of certain light materials and conclusions difficult to predict theoretically.

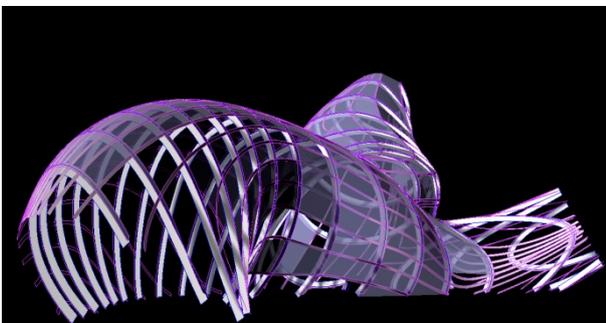


Fig. 16. Graphical resolution of a full model, structure and skin enclosure using developable surfaces for both elements

The use of apparent contours is not limited only to the generation of the developable surface strips that overlay the form. Have been found curves that are closely tied to these contours since geometric point of view and pathways have been developed that rely on these

properties and the approximation to allow the resolution of quadrilateral flat facets or flat panels with utility in the construction of rigid materials that cannot be curved or this process is difficult enough, such as glass.

It is also important to emphasize the possibility offered by the method as to expose the possibility of raising developing lines regarding the geometric resolution of the structure not only the skin also through developable surfaces which meet the requirement of formal consistency. The proposed method presents very versatile for this purpose, in the same way that we solved both solutions faceted surface or flat strips or both solutions combined on a single model, we propose various types of structural design solution of a natural without geometric contradictions.

The extension of this article shall prevent the exposure of the entire system, the exhibition of the rest of results due to this investigation line will be matter of future articles.

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