Modelling of the crumpling process of a paper sheet

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Abstract

The packaging sector needs innovative solutions to face both the worldwide competitiveness between companies and sustainable development issues. In response to this demand, this paper proposes the study of a new folding technique which erects 3D structures from a single flexible sheet. This technique consists in creating a network of complex patterns according to a crumpling process that generates random folds. In order to give a better understanding on the crumpling process, a descriptive modelling is proposed. It highlights useful and useless characteristics conferred to a flexible sheet and gives ways for the development of graph based methods more adapted to the modelling of crumpled structures. A study case on a food packaging supports the proposition.

1 Introduction

Sustainability and climate change are gradually becoming a core factor in managing companies. SME's in particular are conscious of the growing importance of taking into account environmental considerations into their development strategies. Packaging industries do not escape the rule. All around the world, public authorities propose certification schemes for developments in product carbon footprinting and carbon labeling to face the environmental challenge [1-3]. In Sweden for example, the climate certification highlights packaging as an important criteria for the food chain, it aims at raising awareness and stimulating development and production with lower climate impact [4]. Nowadays, packaging systems must be designed with care, using the least amount of materials/energy, maximizing recycled content, and increasing the potential for reuse [5-6]. Classical packages offering only passive, protective functions are no longer adequate. This trend results in the development of advanced active and intelligent food packaging systems that are more dynamic and functional than the classical counterparts [7-9].

The understanding of the ability of a structure to be folded and unfolded is a major issue for the packaging sector. It has been largely covered during the last decade by numerous researchers. Efficient algorithms have been proposed to give best ways to fold maps [10], or to fold tray cartons [11] according to graph based methods. Other approaches dedicated to the mechanical understanding have been also developed. Finite element framework can be used to predict and understand the behavior of a materiel during the folding process [12] as well simplified buckling beam approaches [13]. Experimental studies are also used to understand failures in folded structures as for the factors affecting cracking in a coated layer [14]. Geometric and topological approaches are more in adequacy to understand the folding process in terms of operational sequences. Constraint-based techniques can facilitate the initial design investigation of folded objects and can optimize the folding process by eliminating overlapping of surfaces [15]. Topological information is also powerful to understand the (un)folding process of origami. In this way, graph based methods are used, such as Face Adjacency Graph (FAG) [16] or labelled hypergraph [17]. Finally, mathematical approaches give complementary understanding in terms of computational geometry. First investigations on curvature and creases have been performed in [18]. It gives strategies to open new ways in modelling folding process and folded products [19-21]. A rigorous review of algorithms and computational complexity results about geometric folding and unfolding can be found in [22].

To accompany the global trend for new smart packages, this paper proposes to explore the potential of innovative folded objects based on crumpling sequences. A geometric framework of crumpled objects is first investigated. It gives a typology of crumpled surfaces and their associated operational crumpling sequences. Based on this framework, a descriptive modelling of the crumpling process is proposed and applied on a food package. The understanding of the process is then discussed by highlighting the limitation of existing graph based methods used for the modelling. The conclusion details the perspectives of future works and the applications for packaging sector.

2 Crumpled structures

2.1 Crumpled paper sheet

Folded structures occur in the everyday life at all physical scales. Starting with the DNA structure or a leaf in a bud through the study of orogenic belts, fold is a recurrent physical phenomena. Inspired by Paul Jackson's definition of basic crumpling methods [23], crumpling is a method derived of the origami folding techniques [24]. It consists in a systematic random generation of folds from a single paper sheet in order to create a three dimensional structure. This technique has been used and developed since over fifteen years by designers from CRIMP to create a wide range of biomimetic models as illustrated in fig. 1. This technique creates organized crease patterns very similar to those observed in nature [25-27].

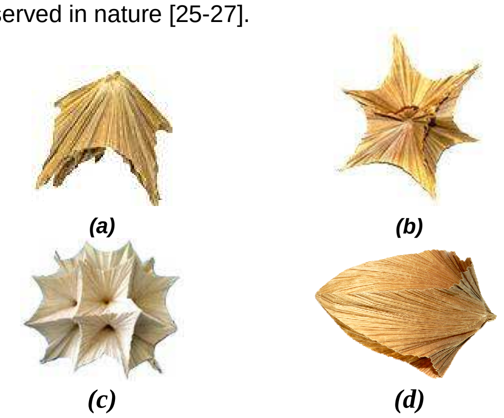


Fig. 1 Crumpled paper sheets.

A crumpled paper sheet contains a network of crease patterns made of folds delimited by ridges. Folds providing from a crumpling process are not individually created by the operator, they are randomly generated by operating pressures on the whole surface of the sheet. Based on CRIMP feedback experiences, three categories of crease pattern highlight: random (the direction of the creases doesn't follow an ordered distribution), parallel (the creases follow a linear direction), and radial (the creases point towards an origin point). The resulting pattern can be formalized by the quadruplet (distribution, origin point, density, geometry) .

2.2 How to crumple a paper sheet?

By comparison to classical origami techniques, crumpling produces creases in a delimited area and are not necessarily based on straight lines. This distinctive characteristic has been early used for wedging and cushioning applications. Many patents [28-29] describe mechanical processes with rolls to transform flat paper into cushions. Such machines generate random folds that transform a flat paper into a flexible three-dimensional structure. A prosperous industry of paper cushioning machines has been generating. The last generation of machines [30] creates volume cushions from flat paper that has been cut to create hanging inside the paper. The added value of such structures only concerns cushion efficiencies, it is not sufficient to achieve the need of new smart packaging which requires the exploration of new way to fold flexible sheet.

CRIMP designers give answer by proposing a crumpling technique which transforms a flat paper sheet into a three dimensional coherent crumpled structure. The process is made of repetitive handcraft sequences of manual pressures. Empiric experiences performed by those designers show that crumpled folds networks can be subdivided into three main categories: concentric pressure, uni-directional pressure and reversal of surface. A concentric pressure applied on a pre-folded paper creates a density of folds pointed towards a targeted point (origin point), the resulting shape is conical (fig. 2a, 2c and 2d). An uni-directional pressure applied on a pre-folded paper generates ridges and/or parallel folds (fig. 2b). The reversal of surface consists in turning inside out a mount (fig. 2c) or valley surface which respectively becomes a valley (fig. 2d) / mount surface.

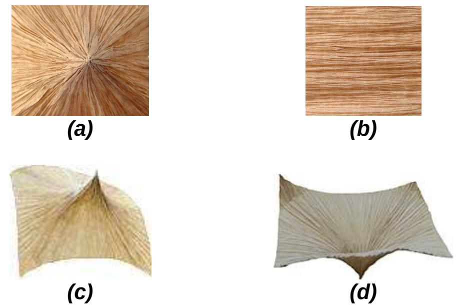


Fig.2. Examples of crumpled surfaces.

The sequences of actions that have been experimented by CRIMP are:

- Concentric pressure/Uni-directional pressure.
- Concentric pressure/Reversal of surface.
- Uni-directional pressure/Concentric pressure.
- Uni-directional pressure/Reversal of surface.
- Reversal of surface/Concentric pressure.
- Reversal of surface/Uni-directional pressure.

From a practical point of view, the reversal of surface often follows a concentric pressure. This repetitive action increases the density of folds near the origin point (in the case of a radial pattern) on the whole surface of the model, it generates a mechanical hanging among the folds and confers strength to the three-dimensional structure as shown in fig. 1b and 1d. As a result, the mechanical performances of crumpled structures release in the density of creases which brings elasticity property to the model. Fig 1a and 1b illustrate the elasticity of an object which can take two geometric configurations. This is a brand new advantage for the packaging sector. The elasticity brings the ability of a product to be stored in a minimum of space; it can also take various configurations depending on requirements during its life cycle, as for its adaptability in size to fit various products to be packed.

3 Descriptive modelling of the crumpling process

3.1 Application to a flexible packaging

Packages fall into three categories: flexible, semi-flexible or rigid. Flexible packaging is considered as the most source-reduced form of packaging. Made of flexible or easily yielding materials, its shape can be readily changed, providing a simple and adaptable answer to portioning, preservation and convenience demand. It uses the least amount of material compared to other forms of packages that could be used for the product. A flexible package adds little weight to the product and leaves little to discard when it is empty; it is considered as the Perfect Fit in the report of Flexible Packaging Europe (FPE) [31].

All those above characteristics can be applied to a three-dimensional structure made of crumpled paper. Indeed, the assembly of crease patterns offers a mechanical flexibility that allows a dynamic change of shapes and can fit to different sizes of products to be packed. A crumpled structure made of paper is considered as a flexible packaging.

The flexibility performance is illustrated in fig. 2 as the ability to be stretchable between the storage phase of the product and its operational configuration. Fig. 2a represents the packaging folded in the hand, and fig. 2b is the same packaging in its operational phase. The resulting crumpled structure is made of a regular assembly of radial crease patterns distributed in a regular manner. The conical shapes of the crumpled surfaces allow the storage and the separation of fruits and can be used as a protection to absorb shocks.

(a) *(b)*

Fig. 3 Two configurations of a crumpled packaging.

3.2 Context and goal of the modelling

The crumpling process is currently performed by the CRIMP designers within a handcraft method. The modeling is then driven by the need for industrial feasibility assessment. Indeed, the competitiveness of the packaging sector enforces reliable industrial processes to manufacture a large amount of products in a minimum of time. Moreover, health feedback from the operators show that after large sequences of manual crumpling operations, elbow tendonitis and carpal tunnel problems offen occurs, what is incompatible with mass production.

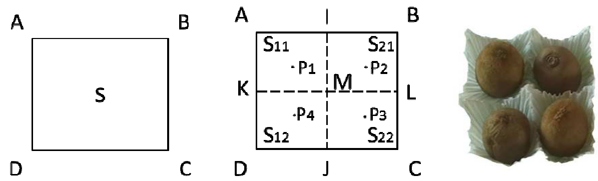
The goal of the modelling is to understand the fundamental principles that allow the technical transformation of a flat material into a complex 3D structure. The understanding of the handcraft transformation will serve as the definition of the basic operations which could be implemented in industry. As a result, it could be possible to manufacture finished products in a sequential manner much faster than with handcrafting-type methods. Mathematical and numerical modelling are not considered in this study.

3.3 Descriptive modelling

The descriptive modelling of the crumpling process identifies the real operations performed on the paper sheet. The boundary of the modelling starts with the definition of the initial and the final configurations of the paper sheet. All operations included between those configurations represent the intermediate states.

The initial configuration represents the product before the first transformation. It is illustrated in fig. 4a by a flat paper sheet S defined by four vertices (A,B,C,D).

The final configuration (fig. 4b) represents the product after the last transformation. The surfaces S_{11} to S_{22} and the ridges IM, MJ, ML, MK result from the folding process. The points P_1 to P_4 are the peaks of the valley surfaces resulting of the concentric pressure of the surfaces S_{11} to S_{22}. The resulting surfaces contain radial crease patterns which can be used to store food (fig. 4c).



(a) *(b)* *(c)*

Fig. 4 Initial and final configurations of the paper sheet.

The intermediate configurations represent all transformations performed by the operator from the initial state to reach the targeted final state. Fifteen intermediate configurations are listed below.

N°	Actions	Conferred characteristic(s)
1	flat paper sheet on support	**Initial state =** flat paper sheet
2	A, D are immobilized	Degree of freedom of A, D = 0
3	Corner B is moved towards the opposite corner A	Movement of the B (speed, acceleration)
4	B is immobilized on A	Degree of freedom of B = 0
		Corners B and A joined
5	border (AB) joined by uni-directional pressure	Creation of the vertex I
6	A, B and I are immobilized	Degree of freedom A,B, I = 0
7	Uni-directional pressure from I to its opposite side (J)	Creation of ridge IJ
		Corners C, D joined
		S is subdivided into S_1 and S_2
8	A,B are immobilized	degree of freedom A,B= 0
9	C,D moved towards A,B	Movement (speed, acceleration) of C,D
		Pre-form of a ¼ folded sheet
10	uni-directional pressure from A,B,C,D to K,L	Location of the starting points K and L of the ridges KM and ML
11	A,B,C,D,K,L immobilized	Degree of freedom = 0
12	Uni-directional pressure from joined points KL to opposite side M	Ridges KM and ML
		Points I, J joined
		S_1 and S_2 are respectively subdivided into S_{11},S_{12} and S_{21}, S_{22}
		S_{11},S_{12}, S_{21}, S_{22} overlapped
13	Immobilization of P	Degree of freedom = 0
14	A,B,C,D,I,J, K,L, M moved towards the same	Movement of the points
		Pre-form of the crumpled structure
15	Concentric pressure of the pre-form among the axis made of P and joined points ABCDIJKLM	Origin points
		Radial crease patterns
		3D conical shapes
16	Opening of the sheet, revert to state 1	Movement of the surfaces
		S_{11},S_{12}, S_{21}, S_{22} not overlapped
17	Reversal of (radial, in, D, mount) into (radial, in, D, valley)	Movement of the surfaces
		Final state = crease patterns network made of regular (Radial, In, D, Valley)

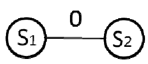
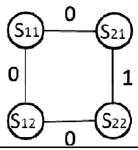
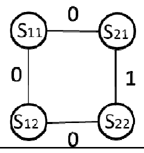
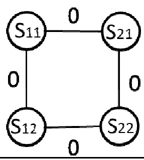
Tab. 1 Intermediate configurations.

The above description only refers to a simple crumpled structure. In order to obtain the targeted strength provided by the folds density, a repetitive sequence of reversal of surfaces must be applied. The quantity of reversal operations is currently evaluated from a qualitative view point by the CRIMP designers depending on their experiences. It is also important to note that peaks P_1, P_2 P_3 and P_4 have been simultaneously created by a morphologic copy during the concentric pressure of S_{11}, S_{12} , S_{21} ,S_{22} which have been previously overlapped.

3.4 Analysis of the crumpling process

The descriptive modelling illustrated in tab. 1 serves at the identification of the useful and useless conferred characteristics depending on their real added value. A useful characteristic brings at least one expected characteristic included in the final configuration of the crumpled structure. A useless characteristic depends only on the technical choice of the process made by the operator, it doesn't give any desired characteristic needed in the final state.

Five useful characteristics have been identified from the actions 7, 12, and 15. They are all related to concentric and uni-directional pressure and reversal of surface. Tab. 2 lists the useful states and shows its associated Attributed Adjacency Graph (AAG) [32-33].

N°	Useful characteristics	Associated AAG
1	**Initial state** = flat paper sheet	
7	Creation of ridge IJ	
12	S_1 and S_2 are respectively subdivided into S_{11},S_{12} and S_{21}, S_{22}	
	Creation of ridges KM and ML	
15	Origin points	
	Radial crease patterns	
17	**Final state** = crease patterns network made of regular (Radial, In, D, Valley)	

Tab. 2 Useful intermediate states.

All other useless conferred characteristics depend on the technical choices of the actions made by the operator. Actions 2,3,4,6 for example give characteristics as for speed or immobilisation which are not expected in the final configuration. More important, action 12 provides 4 characteristics but only 3 are kept. The useless characteristic "S_{11},S_{12}, S_{21}, S_{22} overlapped" is not a useful added value expected in the final configuration. In this case, a new process should be proposed in order to only obtain the useful characteristics. In terms of value engineering, all useless characteristics providing from actions could be suppressed or reduced to optimize the industrial process. Remember that tab. 1 describes the current crumpling process, other ways to proceed could be possible. Based on the useful added characteristics (tab. 2), industrial engineers should focus on the best solutions which will meet both environmental and economic requirements.

4 Discussion

The descriptive modeling gives an operational understanding of a crumpling process. It also gives an analysis of the conferred characteristics on the product by highlighting useful and useless actions. Additionally, the modeling with the Attributed Adjacency Graph only gives some geometric and topological knowledge. The use of a revised Extended Attributed Adjacency Graph [34-35] can give complementary information to fit the specific topological characteristics of a crumpled object. However, such objects are not made of planar surfaces and straight edges. The sequences which create a crumpled object differ from classical manufacturing processes based on milling for example. CAD/CAM Softwares which have been developed based on attributes defined in EAAG are consequently not in adequacy with crumpled objects. New algorithms must be created. In this way, the development of hypergraph methods must be performed. Their efficiency has been recently proved for classic origami [17], their adaptation on crumpled objects must now be engaged.

5 Conclusion

Crumpling is a new origami technique which generates complex crease patterns from a single paper sheet. In order to understand the transformation of a single paper sheet into a 3D structure, a descriptive modelling of the crumpling process is proposed. After the analysis of all actions of the process, some useful characteristics conferred to the flexible sheet, shows that three fundamental actions are required in the process. They consist in unidirectional pressure, concentric pressure and reversal of surface. By combining different operational sequences of those actions, various complex structures can be created. A complementary modelling of the topological characteristics of the crumpled model within a graph based method gives a better understanding but also proves its limitation. The use of hypergraph will give complementary information and will be discuss in another paper. Additional researches will be also conducted to understand the crumpling mechanical performances of the crumpling process applied on various materials. This work will serve at the delimitation of the implementation of crumpled structures in the packaging sector.

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