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Masonry Vaults: The efficiency of geometry

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Abstract

Purpose:

The present original paper reports on an investigation on vaulted ceilings aimed at the –digitally supported- search of the relations between the geometry of such architectonic elements and their structural behaviour. It could therefore be categorized as 'Product Geometrical and Functional Characterization'.

Method:

A search for the balance of forces acting through these constructive elements is propounded, upholding on the Limit Analysis Theory. Graphically, the element's security limits are evaluated according to the forces' trajectory distance through the vault or dome shapes.

Result:

A simulation of the application of this graphical method on three equally dimensioned square-based types of vaults is presented, namely the barrel, the groin and the dominical vault. In the light of the results obtained, the structural characteristics of each vault are evaluated.

Discussion & Conclusion:

The results prove the feasibility of the method as well as the extrapolation of values to any other reasonable dimension within the constructive resources considered. The information displayed in this article is part of a larger survey that further studies a higher number of domes and vaulted ceiling typologies. Nevertheless, such results are regarded to be sufficiently representative of the whole analysis. In particular, weights and thrusts performed by the vaults on the walls that support them, stresses and their distribution on each geometry configuration are obtained from this survey. The strained joints are also detected, measured and evaluated, in order to set a limit for pathologic deformations according to support displacements.

1 Introduction

This survey addresses the close relation between the geometry of masonry vaults and their structural behaviour. Arches and domes are considered a type of vault, thus all the vaulted structures are taken into consideration. All of them are masonry structures, made of a discontinuous isotropic material, either stone or brick-made, and formed by a group of small elements when compared to the global structural dimensions.

The abovementioned construction elements are very common in the region of Extremadura, and are mainly linked to monumental purposes as well as to residential and agricultural applications. The vaulted constructions throughout the region are typically featured by a singular construction technique: they are built without falsework, that is, without an auxiliary structure helping throughout the construction process. This particular attribute sets Extremadura and the Portuguese area of Alentejo as worldwide references.

New technologies applied to construction techniques derived from the industrial revolution, such as steel and concrete, have pugnaciously and arrogantly monopolized the market, in a similar way petroleum did as referred to the energy industry. However, a new interest for masonry has been seen to emerge in the last decades, undoubtedly influenced by the late increase of restoration and rehabilitations, activities with growing relevance due to the economical crisis. Provided this type of structure works at low stress regimes, its frailness is not conditioned by material resistance, but by spatial configuration, shape and geometry.

The present study is aimed at performing a structural analysis of masonry vaulted constructions and, more precisely, at setting a correlation between resistance and geometry.

In order to do so, a structural behaviour analysis of these masonries is proposed, mainly focused on geometrical configurations, to unveil the relation between geometry and acting forces. The stability of these structures relies on its balance, its weight balance, its volumetrics weight, and its combined volume and geometry.

2 Methodology

In order to meet these goals, the implemented methodology consists on a systematic analysis of the arches, domes and vaults' structural behaviour based on the Limit Analysis Theory. Once this methodology is applied to structures with different geometries, results are compared to stress the relation between geometry and behaviour as well as to quantify such degree of correlation.

This methodology is unprecedented. Even though complex brickwork and masonry geometry are traditionally reported to be related to resistance, the methodology involved was based on particular “tricks” to determine the dimensions of elements such as straps or buttresses rather than on specific graphical techniques.

This work is focused on the analysis of a set of arches, domes and vaults classified in the following groups: single curved elements, double curved elements and complex elements. Barrel vaults and their derivatives are included in the former group. The second category stands for radial domes, 'Extremeñan' vaults and its derivatives. Finally, the third category includes ribbed vaults in general, designed with ridge ribs and severies.

Our own structural calculation software, referred to as CARYBO, was used for the structural analysis of the elements. It is registered in the Territorial Registry for Intellectual Property of the Regional Government, with 14/2011/548 record entry and November 18th 2011 effective day, according to application number BA/78/11 as regulated by the Intellectual Property Law (R.D. 1/1996 April the 12th), with Manuel Fortea Luna as transferee title.

Several types of results were achieved for each of the analyzed items, classified by origin and nature of the element. Information was then comparatively assessed in search for constant parameters or behaviour trends.

2.1 Analytical models

2.1.1 Limit Analysis Theory

The first book that rigorously gathers the plastic theory of steel frames calculus was published by 1956: “*The steel skeleton. Vol. 2: Plastic behavior and design*”[1]. It condensates and ads up all the material the Cambridge Team had achieved for the last decade and contains the first reference ever to the Limit Analysis Fundamental Theorems in a calculus publication.

The Fundamental Theorems where demonstrated in 1936 by the Russian engineer Gvozdev, but rather darkly published on the Moscow Science Academy Record (in Russian). They went unnoticed and were rediscovered by Praguer's Team by the early 50s. The application of these theorems to steel frames allowed to set the plastic calculus (which had been used in England since 1948 - the year the British regulations added a clause to account for plastic calculus-) in a precise framework.

Plastic theory was originally developed for steel structures and further discovered to be applicable to reinforced concrete structures. Actually, plastic theory is suitable for any ductile structure which is proved to show no instability. This fact, guessed by the early 20th century engineers, was clearly and accurately stated by Professor Heyman.

The theory was immediately applied, although with certain limitations, to reinforced concrete frames. In the 60s Jacques Heyman realized that the same theory could be applied to masonry and brickwork structures. His job added theoretical rigor to an area that had remained stagnant since the late 19th century.

Professor Heyman was the first to realize that the Fundamental Theorems constituted a new paradigm that could be applied to any structure built with conventional materials.

This might seem evident for reinforced concrete elements (in fact, Gvozdev's contribution in 1936 was focused on the determination of limit loads for reinforced concrete structures) but it was not so obvious for the case of materials such as wood, and not evident at all if regarding masonry or brickwork structures.

Heyman guessed that the theorems could also be adapted to masonry and brickwork, and even to wood structures. He proved the new paradigm could be applied to any ductile structure. In fact, the main corollary of the Safe Theorem of Limit Analysis stands for what Heyman refers to as the “equilibrium approach”, by which the analyst shall only use two or three fundamental equations, namely those relating equilibrium and material. His approach to equilibrium has already been used by eminent engineers guided by such structural intuition (Gaudi, Maillart, Nervi...). Even though this is a key theory for any architect or engineer working on structures, it was also seen to be useful for historians working on architecture, engineering and construction, as well as for anyone who would like to deepen on the development of structural forms. In relation to the latter case, Jacques Heyman's studies have decisively contributed to the development of the history of construction techniques.

His manuscript entitled “The Stone Skeleton”[2] was published in 1966, and was a clear and original approach to the adaptation of the plastic theory to the field of traditional masonry construction. Following Praguer's suggestions, he states that the Fundamental Theorems can be translated into these (apparently so different) structural types by merely assigning certain properties to masonry.

After his first article in 1966, more than thirty other ones and several books have followed up to date, in which Professor Heyman applied the modern theory to the study of the basic structural elements of masonry construction (vaults, domes, buttresses, towers, steeples, etc.). In fact, his interpretation of gothic theory yields to conclude the debates on the structural behaviour of the cathedrals studied by theorists since mid 19th century (Viollet-le-Duc, Ungewitter, Mohrmann, Abraham, etc.).

Professor Heyman's contributions to the field of masonry vaulted structures are pivotal. In fact, it would be hard to picture what the state of this discipline could be today without such studies, which had provided architects and engineers with a rigorous theory to face the analysis of historic structures built with long-gone materials and methods.

In the same field of historic structures, Professor Heyman had also proved the Limit Analysis Principle to be a suitable tool for application to wood reinforcement.

The Plastic Theory is based on Gvozdev's Principles, which establish that only three types of equations shall be stated:[3] *(i)* the equilibrium equations; *(ii)* the yield strength conditions (no internal stress must be higher than the material stress limit); and *(iii)* equations to account for the

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[1] *J. F. BAKER, M. HORNE Y J. HEYMAN. The steel skeleton.. Vol. 2: Plastic behaviour and design. Cambridge University Press. 1956*

[2] *HEYMAN, JACQUES. The Stone skeleton, International Journal of Solids and Structures.1966*

[3] *HEYMAN, JACQUES. The Science of Structural Engineering. Imperial College Press, London.1999.*

fact that a deformation mechanism must take place during structural collapse.

Gvozdev proved three theorems based on the application of the three conditions, namely those of Unicity, Insecurity and Security.

The first one is the main one, and reads as follows: “If all conditions are satisfied simultaneously, the collapse load corresponding to the equations’ solution has a defined and computable value”. This theorem confirmed the studies that were being performed by Baker.

The second theorem deals with insecurity: “If attention is focused on the possible collapse mechanisms and equilibrium is not required, and, moreover, the yield strength is not necessarily satisfied on each part of the structure, it is then still possible to calculate a collapse load value”. However, such value is not fully reliable. The designer will believe the structure to be more resistant than it actually is.

Finally, the third theorem constitutes the basis for the current structure calculus theory and might be simply stated as follows: “Whenever the designer can find a configuration ensuring an appropriate behaviour under specific loading conditions, the structure should be regarded as safe”. The strength of this statement lays in the fact that only a single well-behaved configuration needs to be found, which must not necessarily be the actual way the structure behaves, provided that if the designer found one suitable configuration, then the structure itself would certainly approach such configuration as well.

2.1.2 Segment system

The most famous and simple vault is that of semi cylindrical barrel, which is formed by a linear and continuous element whose upper part is a semi cylinder supported on two parallel walls. The dome membrane has a lower surface (intrados), a visible cylindrical shape, whereas the upper surface (extrados) is the hidden side. It is usually more irregularly shaped, and it holds either another floor or the roof. This vault can be dissected into a set of thin parallel contiguous arcs similar to slices. Vault analysis can be simplified by focusing on only one of those small arches and further computing the sum of that and the remaining arches.

Consider a barrel vault sliced into one-meter-width segments. The analysis of one of those arches would yield the results on the vertical and horizontal thrust supports. It might therefore be assumed that these results are coincident to the vault supports for each linear meter. However, an arch is not a one-dimensional element, but it is also described in terms of edge depth and thickness. Three dimensions provide structural volume and, depending on density, net weight (Figure 8, parallel slice barrel vault).

This arch-slicing technique can be extended to any other type of vault. The single groin vault is the intersection between two semi cylindrical barrel vaults. If divided into slices, arches do not rest on the supports as in the former case, but on the groin, merging in pairs, one for each barrel vault. However, the groin cannot be dimensionless if it collects stresses since it would imply receiving infinite stress. Instead, it must be an arch spanning from one groin to the other one, passing through the centre, where it crosses the other groin (Figure 9, parallel slice groin vault). Dominical and ribbed vaults can be segmented in arches which may themselves rest on other arches.

Segments should be symmetrical to the central axe, traversed by the line of force. Coplanar lines of force that are contained in the mid section might act as a symmetry axe. This way, segments can be divided into the three following types:

Parallel-sided segment: Barrel vaults and its derivatives produce the simplest slice, with parallel sides and constant width. It is a section of the vault extended through the third dimension with a constant value for each point. It is a fixed width arch with a central symmetry plane.

Converging-sided segment: Hemispheric domes and its derivatives produce a wedged slice, consisting in a section of a third-dimension extended dome with a variable value. Its width maximizes at its support and minimizes at the keystone, with a similar central symmetry plane. Wedge-shaped converging sides can be decomposed from the fan vault. In this case, the minimum and maximum slice widths lay at support and keystone, respectively.

Non-coplanar axe segment: Spiral staircases and helicoids can be decomposed in segments, but such elements have no symmetry axe, nor they have a symmetry plane. Accordingly they cannot produce coplanar lines of force. Besides the gravitational force and the thrusts, a rare force operates for this particular case, namely the centrifugal force, which sets forces to undergo a path different from that of the maximum slope.

Coplanar axe segments are traversed by a force line. This line is also referred to as the thrust line, and is the geometric place where the resulting force is located within a given system of sectioning surfaces.

Consider now a symmetrical arch for which only a single half is accounted for. For this semi-arch to be balanced under its own weight, a horizontal force in the keystone is needed, which is provided by the other semi-arch. The force line starts right where the other semi-arch’s horizontal force acts. The vertical loads corresponding to each section of the arch add up to such force. This way, the force line is defined as the set of geometric points of the resultant vector on every section of the semi-arch.

2.2 CARYBO. Software for the structural analysis of Arches, Vaults and Domes developed by the author

2.2.1 Background

CARYBO is a digital tool developed by the author for the structural calculation and analysis of masonry and brickwork, including arches, vaults, buttresses, straps, flying buttresses, stairs and walls. Its field of action covers any structural element exclusively subject to compressive forces.

The book “Bóvedas Extremeñas” (*Vaults from Extremadura*) was edited in 1998 by Manuel Fortea and Vicente López Bernal. Its second half displays spreadsheets for 24 vaults, for each of which a basic characterization is performed from simple data inputs. A simple program based on a standard spreadsheet was designed in order to translate the graphic problem into a strictly numerical issue. Such program has been improved from then up to date, and 2011’s Java version was registered as 14-2011-548 code.

2.2.2 Method

The program was designed to graphically find a force line in an arch segment for given section, density and acting loads (security coefficient to meet appropriate range). For such purpose the arch was divided into 200 voussoirs, and a weight was assigned to each one according to its volume and density. Accounting for the symmetry principle, only a single half of the arch must be considered for analysis. For this semi-arch to be balanced under its own weight, a horizontal force in the keystone is needed, which is provided by the remaining semi-arch.

The procedure undergoes both manual and automatic stages: input values are manually introduced, whereas results are automatically computed through suitable analytical expressions. Whenever the force line gets out of the section or the security coefficient decreases below a critical value, the solution shall be discarded and new input values shall be assigned to the variables (either individually or grouped for each step).

Obviously, there will be more than one force line to meet the abovementioned conditions. However, provided energy is not spontaneously wasted, the one with the least horizontal thrust ought to be selected.

2.2.2.1. Voussoirs

The semi-arch can be divided into voussoirs –that is, independent segments-. CARYBO divides the semi-arch into 100 voussoirs cut by vertical plans. The fact that sections are vertical or radial has no relevance for calculation purposes. As shown in figure 1 –Arch voussoirs- each voussoir is defined by extrados and intrados lines, with a constant width of a hundredth of the arch's length.

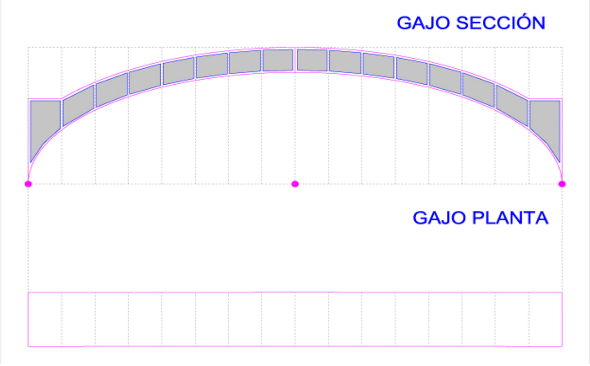


Fig. 1 Arch-voussoirs segment .

2.2.2.2. Force Line

The thrust line starts in the keystone with the force transmitted by the other semi-arch. From this point, the force that each voussoir applies is vectorially added up (considering each net weight and overload). The sequence of this vectorial set is the thrust line or Force Line, which eventually reaches the support, with horizontal (the one initially applied to the keystone plus punctual horizontal overloads if any) and vertical (own weight, overloads and punctual loads if any) components. The point A_C in figure 2 –Force Line arch- represents the point of application of the force Eh due to the other semi-arch, while point R represents the end of the Force Line at end of the arch.

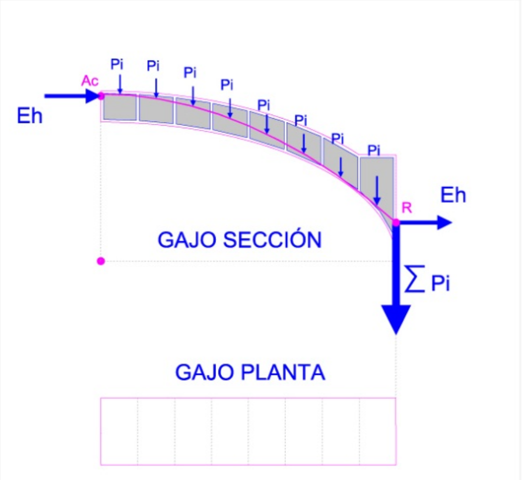


Figure 2 Arch-Force Line Segment.

2.2.2.3. Variables

Out of the elements intervening on the Force Line, weights and loads are input data. Thrust Eh due to the other semi-arch and point of application Ac are variables. This way, the modification of the value Eh or the height of Ac leads to the generation of a new Force Line. In any case, this Force Line must be contained within a fringe inside the segment's section, determined by the previously set Geometric Security Coefficient. If the thrust line would get out of the limits, any of the variables could be altered (the horizontal thrust on the keystone or the application point), or even both, until a thrust line that meets the stated conditions is obtained. Out of all the possible Force Lines that satisfy the desired conditions, the one with a lowest horizontal thrust ought to be chosen. In figure 3 -Arch Force Line Set- the one undertaking on point Ac1 with associated thrust E1 triggers a reaction on point R1 with magnitude represented by Et1. The one undertaking on point Ac2 is associated to force E2, and reaches the end of the arch on point R2 with intensity Et2. For this particular type of arch the higher the location of the point of attack on the keystone of the horizontal force, the higher the intensity of the thrust at the end of the element. The opposite applies to stilted and pointed arches.

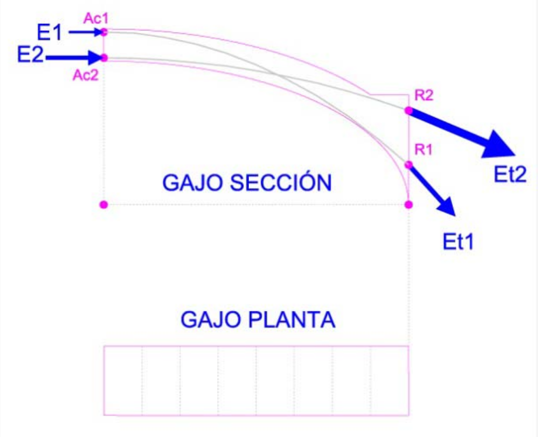


Figure 3 Arch Force Line Set segment.

2.2.2.4. Geometric Results

Once the arch intrados and its width are known, the intrados surface can be straightforwardly computed. As can be observed in figure 4 -Surface arch-, the length of the curved line ACB times the width d equals the intrados surface, which can also be mathematically obtained by summing up the lower surfaces of voussoirs. The so obtained intrados surface is an “actual magnitude”, which is an important fact for the builder who wishes to quantify the material needed for the construction of the arch. This issue was thoroughly investigated elsewhere by authors like Hernán Ruiz *et al.*, whose solution was computed graphically. If this surface as real-dimension arch was divided by the plant’s projection surface, a one-dimensional value known as Concavity Coefficient would be obtained. This parameter would equal unity and π/2 (and even higher for pointed arches) for a completely flat surface and for a circular arch, respectively. This way, the Concavity Coefficient is linked to the concavity of the arch as well as to the distance the keystone is from the vertical starting pieces. Values below/above π/2 represent basket-handled/ pointed or stilted arches, respectively.

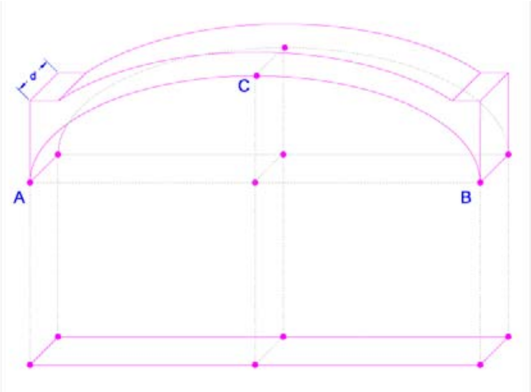


Figure 4 Surfaces Arch segment.

2.2.2.5. Results for total loads

Total loads are known, since they are computed as the sum of all the external loads and those generated by the own net weight. If this value was divided by the arch plan surface, the total (horizontally projected) load per unit area would be obtained. This value referred to the arch’s net weight is also interesting for builders because it will allow them knowing how much material they would need for the construction of the whole arch, including the sinus filling.

Results for support reactions:

The Force Line ends in the arch extreme on point R (figure 5, Arch-Support Reactions) at height Yr as referred to the arch starting point, with total intensity Er and inclination angle a. The arch will be supported by an element such as a wall or a column. This will be the result used in the support analysis, keeping in mind that it should meet this action for that particular value and for that specific point of application.

The resultant Er has two components. A vertical one, which equals the net weight of the arch plus vertical overloads, and a horizontal one, which depends on equilibrium conditions. The lower the arch’s horizontal component is, the more stable it will be. Also, the lesser the horizontal component is, the lower the point of application R will be located.

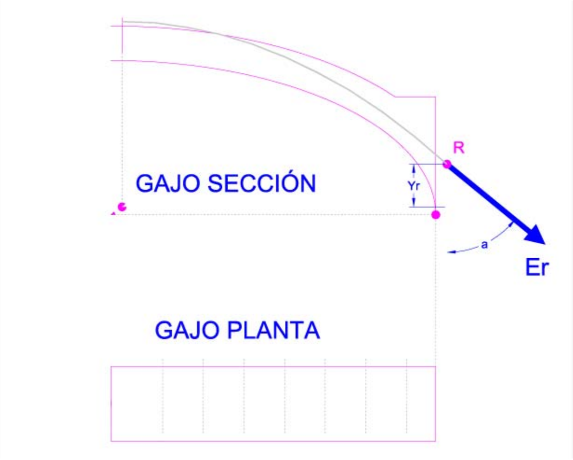


Figure 5 Arch Segment support reactions.

2.2.2.6. Results: articulation points

Prior to collapse, the points where the articulations will take place are the same for which the force line gets closer to the section limits (figure 6, Arch Segment Articulations). Articulations are the points where the arch breaks to turn into a mechanism. When arch supports suffer small displacements, cracks coincident with the opposite side articulations will appear. Intrados articulations will provoke cracks in the extrados and vice versa.

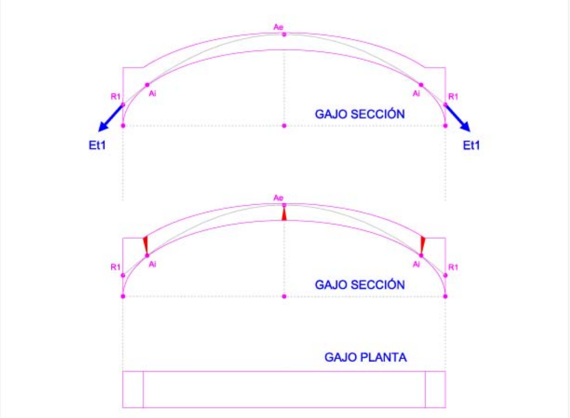


Figure 6 Arch Segment Articulations.

Collapse displacements:

Three articulations are at least needed for collapse to take place. If a shift in the support location is supposed, then the arch is divided in four elements which are linked by the articulations, which also operate as hinges. When the arch elements “have enough space” to fall, which will certainly do, collapse will take place (figure 7 Arch Segment Collapse). At this moment the three articulations are horizontally aligned. Collapse span is the arch span plus 2xAC(1-cos(a)), meaning twice the distance between articulations (AC) multiplied by one minus the cosine of the angle formed by the line joining two consecutive articulations and the horizontal line.

This is the theoretical instant at which collapse is expected to take place, but it actually does before that moment, when corner articulation points break. However, collapse span values are useful to know how close an arch stands from its critical state. For instance, the closer the distance between articulations, the lower the collapse span; whereas the higher the distance between articulations, the higher will be the shifts needed between supports in order to reach collapse.

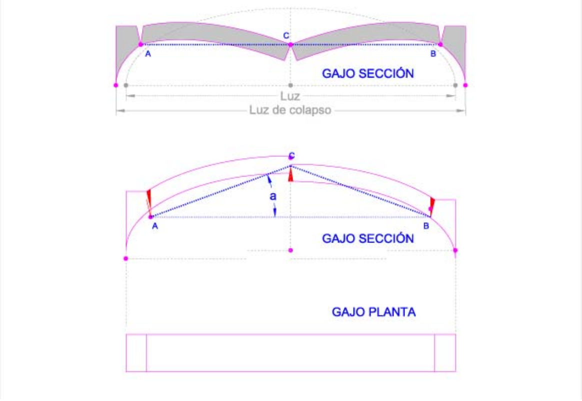


Figure 7 Arch Segment Collapse.

2.2.2.7. Results: Safety Coefficient at each point

The Geometric Safety Coefficient (GSC) is a one-dimensional value that indicates for each voussoir thrust location relative to the limit. A minimum value of 1.1 has been set, which implies being within a 90% of the section. Points with GSF equal to 1.1 coincide with articulation points. The higher the GSC, the closer to the section centre the Force Line will be. Ultimately, GSC values indicate the location of the Force Lines within the arch.

2.2.2.8. Stresses on each point

When the strength traversing each voussoir is known, its stress value can be obtained by simply dividing such strength by the voussoir section. Such values are usually way below the admissible material stress and are therefore not usually decisive when testing the arch's stability.

2.3 Set of parallel-sided segments

2.3.1 Barrel vault

A barrel vault is constituted by simply a series of arches located one next to each other. From a structural point of view, the analysis of a single vault segment is enough to achieve all necessary information. For the analysis of a barrel vault, it shall be sectioned in a series of vertical planes parallel to each other within a given constant distance. One single segment (an arch) should be analyzed to further export output data to the complete vault.

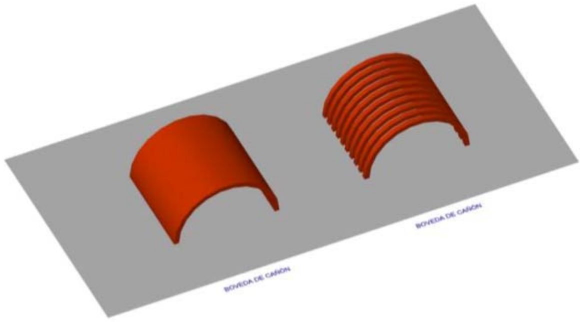


Figure 8 Barrel Vault Segment.

2.3.2 Groin vault

A groin vault is defined by the intersection of two barrel vaults. It should be divided into a set of arches in order to undergo an appropriate analysis. Firstly, two arches corresponding to the diagonals between opposite corners, crossing at the keystone, should be considered. Secondly, each of the four barrel vaults will be divided in arches. These vault's arches span will be lower as they reach the keystone, and they are not supported by walls but by the diagonal arches. Every arch, or portion of arch, should be analyzed independently according to its specific geometry and load. Diagonal arches will be analyzed keeping in mind that they are supporting the corresponding barrel vault's arches, therefore receiving every stress transmitted by them

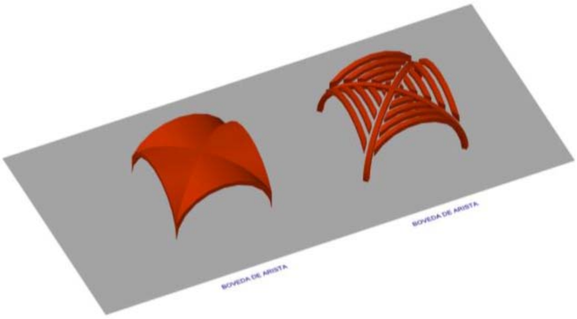


Figure 9 Groin Vault segment.

2.3.3 Converging-sided segment.

Jack Heyman pointed out the most common of dome cracking processes, which is indeed easy to observe: just take the already pressed half of an orange, put it on a table and push it until cracks start to appear. As shown in figure 10, radial cracks of a certain length (the top not being reached) will appear.

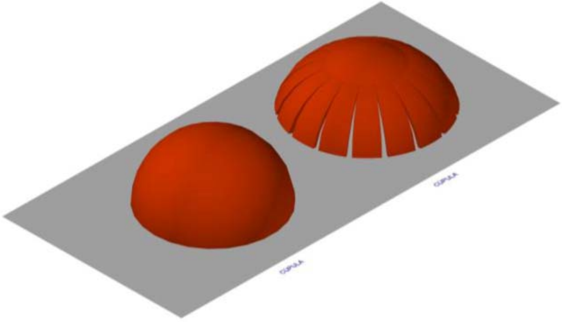


Figure 10 Dome cracks.

When dividing a dome, the obtained segments will not show parallel sides, as in barrel vaults. In this case a revolution surface will be generated, and therefore wedge shaped segments (narrower at the keystone and wider at the base) will appear when decomposed.

Figure 11 Dome segments.

The way the dome cracks indicates which sections are independent in extreme situations, as it happened in the groin vaults. Structurally, the dome is divided in independent elements that work separately. Cracks set the limits among segments. Of course, only segregation cracks are referred to, and not articulation ones. Such cracks can be noticed both at the intrados and the extrados. No stress takes place on them, not even in a single point, as it happens in articulation cracks. If Heyman's deformed dome is divided in segments, matching every crack with its corresponding element, the next situation is obtained.

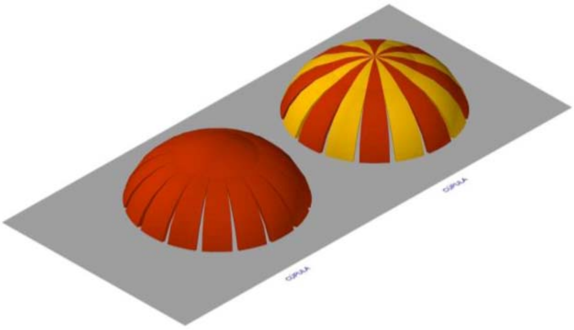


Figure 12 Deformed dome segments.

The difference among barrel vault segments and revolution dome segments lies in the position of its outer sides. Barrel vault segment sides are parallel, and dome segment sides converge to the highest point (keystone).

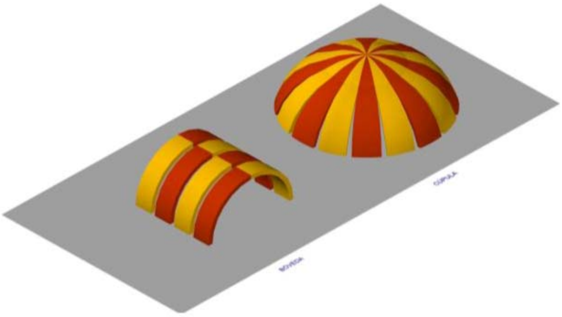


Figure 13 Vault and dome segments.

If a dome segment is isolated, it is noticeable that it is self-stable since it is balanced by its opposite half segment. However, dome segments, corresponding to dome semi-sections, are not balanced by collateral ones. This reasoning is self evident when considering a dome with a central oculus, where no contact between opposing segments exists.

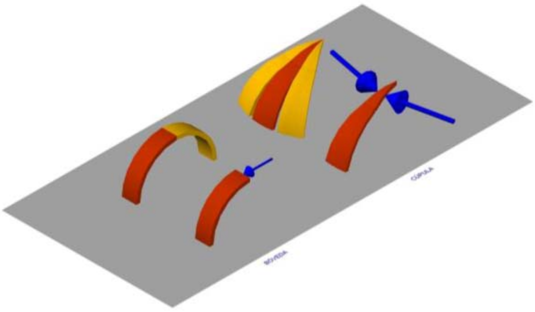


Figure 14 Dome and Vault semi-sections.

If a semi-section is accounted for, the supports ought to be substituted by the forces they are applying if equilibrium needs to be maintained. In the case of the dome, the force is applied by the opponent semi-arch. However, in the case of vaults, the forces involved are those applied by the collateral semi-sections.

2.3.4 Domes.

Domes are geometrically defined as the revolution surface generated by the spin of a curve around a vertical axis which is located on the highest part of the curve. This way, a sphere might be regarded as a dome whose generating curve is a circumference with an axis situated on the circumference's upper quadrant.

2.3.5 Fan vaults.

Fan vaults are geometrically defined as the revolution surface generated by the spin of a curve around a vertical axis, situated on the lower part of the curve (instead of the upper part, as applies for the case of domes). The dome and the fan vault sections are therefore inverted as compared to each other.

2.3.6 Horizontal toroid vaults.

Toroid vaults are geometrically defined as the revolution surface generated by the spin of a curve around a vertical axis located at a certain distance from the curve.

2.3.7 Non-coplanar segments.

Geometrically, a spiral stairway is a helicoid. This surface cannot be dissembled in coplanar segments, i.e. segments whose axes are contained in a plane. It can be dissembled in non-coplanar segments and cannot therefore generate a coplanar Force Line. In this case, not only the gravitational force and the thrust are acting, but also a rare one, the centrifugal force, which will compel the gravitational forces to travel a path that doesn't match the steepest line.

3 Conclusion

3.1 Vaults

Aided by CARYBO software, three square-based vaults were analyzed, each one with a different geometrical section. The first one is a barrel vault, the second is a groin vault and the third one is a dominical vault. Plan dimensions and loads were set as constant values.

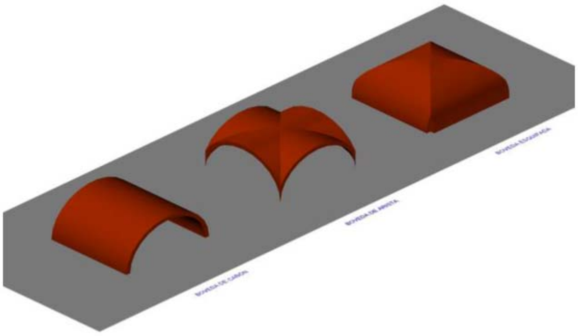
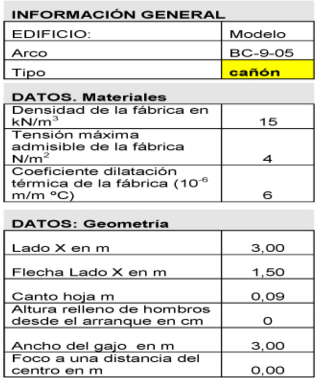


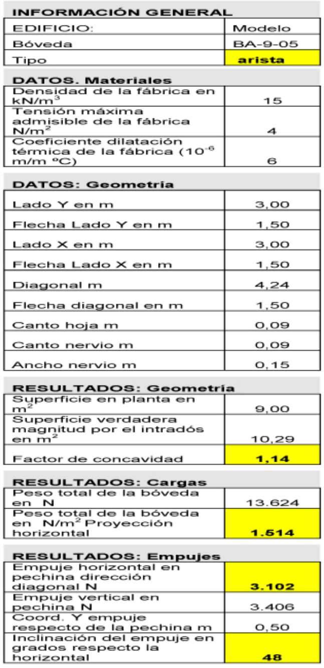
Figure 15 Barrel vault, groin vault and dominical vault.

Each of these three types of vaults shows a different structural behaviour. The results achieved for the three vaults are shown below. Note they are 3 metre long squared-based with 9 centimetres sheet edge, keystone located at 1.5 metres high and 15 kN/m^3 material density.

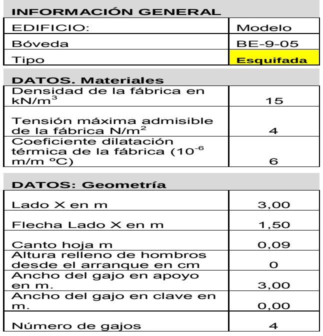


INFORMACIÓN GENERAL	
EDIFICIO:	Modelo
Arco	BC-9-05
Tipo	**cañón**
DATOS. Materiales	
Densidad de la fábrica en kN/m^3	15
Tensión máxima admisible de la fábrica N/m^2	4
Coeficiente dilatación térmica de la fábrica (10^{-6} m/m ºC)	6
DATOS: Geometría	
Lado X en m	3,00
Flecha Lado X en m	1,50
Canto hoja m	0,09
Altura relleno de hombros desde el arranque en cm	0
Ancho del gajo en m	3,00
Foco a una distancia del centro en m	0,00

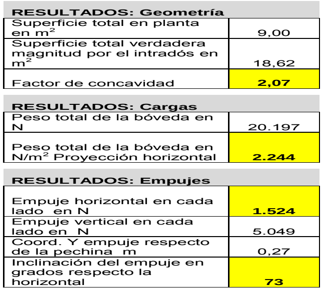
RESULTADOS: Geometría	
Superficie en planta en m^2	9,00
Superficie verdadera magnitud por el intradós en m^2	14,12
Factor de concavidad	**1,57**
RESULTADOS: Cargas	
Peso total de la bóveda en N	16.848
Peso total de la bóveda en N/m^2 Proyección horizontal	**1.872**
RESULTADOS: Empujes	
Empuje horizontal en apoyo en N	**4.610**
Empuje vertical en apoyo en N	8.424
Coord. Y empuje respecto de la pechina m	0,47
Inclinación del empuje en grados respecto la horizontal	**61**



INFORMACIÓN GENERAL	
EDIFICIO:	Modelo
Bóveda	BA-9-05
Tipo	**arista**
DATOS. Materiales	
Densidad de la fábrica en kN/m^3	15
Tensión máxima admisible de la fábrica N/m^2	4
Coeficiente dilatación térmica de la fábrica (10^{-6} m/m ºC)	6
DATOS: Geometria	
Lado Y en m	3,00
Flecha Lado Y en m	1,50
Lado X en m	3,00
Flecha Lado X en m	1,50
Diagonal m	4,24
Flecha diagonal en m	1,50
Canto hoja m	0,09
Canto nervio m	0,09
Ancho nervio m	0,15
RESULTADOS: Geometria	
Superficie en planta en m^2	9,00
Superficie verdadera magnitud por el intradós en m^2	10,29
Factor de concavidad	**1,14**
RESULTADOS: Cargas	
Peso total de la bóveda en N	13.624
Peso total de la bóveda en N/m^2 Proyección horizontal	**1.514**
RESULTADOS: Empujes	
Empuje horizontal en pechina dirección diagonal N	**3.102**
Empuje vertical en pechina N	3.406
Coord. Y empuje respecto de la pechina m	0,50
Inclinación del empuje en grados respecto la horizontal	**48**



INFORMACIÓN GENERAL	
EDIFICIO:	Modelo
Bóveda	BE-9-05
Tipo	**Esquifada**
DATOS. Materiales	
Densidad de la fábrica en kN/m^3	15
Tensión máxima admisible de la fábrica N/m^2	4
Coeficiente dilatación térmica de la fábrica (10^{-6} m/m ºC)	6
DATOS: Geometría	
Lado X en m	3,00
Flecha Lado X en m	1,50
Canto hoja m	0,09
Altura relleno de hombros desde el arranque en cm	0
Ancho del gajo en apoyo en m.	3,00
Ancho del gajo en clave en m.	0,00
Número de gajos	4



RESULTADOS: Geometría	
Superficie total en planta en m^2	9,00
Superficie total verdadera magnitud por el intradós en m^2	18,62
Factor de concavidad	**2,07**
RESULTADOS: Cargas	
Peso total de la bóveda en N	20.197
Peso total de la bóveda en N/m^2 Proyección horizontal	**2.244**
RESULTADOS: Empujes	
Empuje horizontal en cada lado en N	**1.524**
Empuje vertical en cada lado en N	5.049
Coord. Y empuje respecto de la pechina m	0,27
Inclinación del empuje en grados respecto la horizontal	**73**

Out of these three vaults, the groin is the lightest (1.514 N/m^2) and the one with the lowest Concavity Factor (1.14). On the contrary, the dominical vault stands as the heaviest (2.244 N/m$^{2)}$ and that with the highest Concavity Factor (2.07). The barrel vault is only supported by its sides, and 4.610 N horizontal thrust was applied on each of them. The groin vault is supported on its four corners (horizontal thrust 3.102 N, diagonally directed). Finally, the dominical vault is supported on its four sides (horizontal thrust of 1.524 N on each).

In absolute terms, stress value on the three of them is low. However two issues must be highlighted: maximum stress on the groin vault is 0.03 N/mm^2, getting quite higher on the edges, up to a value of 0.34 N/mm^2. On the dominical vault the maximum radial stress -that transmitted to the supports- is 0.03 N/mm^2, but a parallel-to-the-supports ring stress appears (with a different value depending on the vertical position in relation to the supports), which maximum value reaches up to 0.17 N/mm^2, way higher than the radial stress. As it applies for arches, maximum work stress is not the defining value in order to guarantee stability for this element.

The thrust tilt of these three types of vaults is visibly different. The most horizontal thrust is produced by the groin vault, with 48º inclination from horizontal plan, whereas the steepest is produced by the dominical vault with 73º inclination.

Because of support displacement, distance between joints is somehow linked to the remaining time until collapse. The closer they are, the faster collapse will take place. Under this enclosure, the safest vault is the barrel, provided its joints are separated 38% of total span. On the other hand, the dominical was regarded as the least safe, as the joints are separated 20% of their total span (although difference is not too relevant).

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